EC8491 - Communication Theory

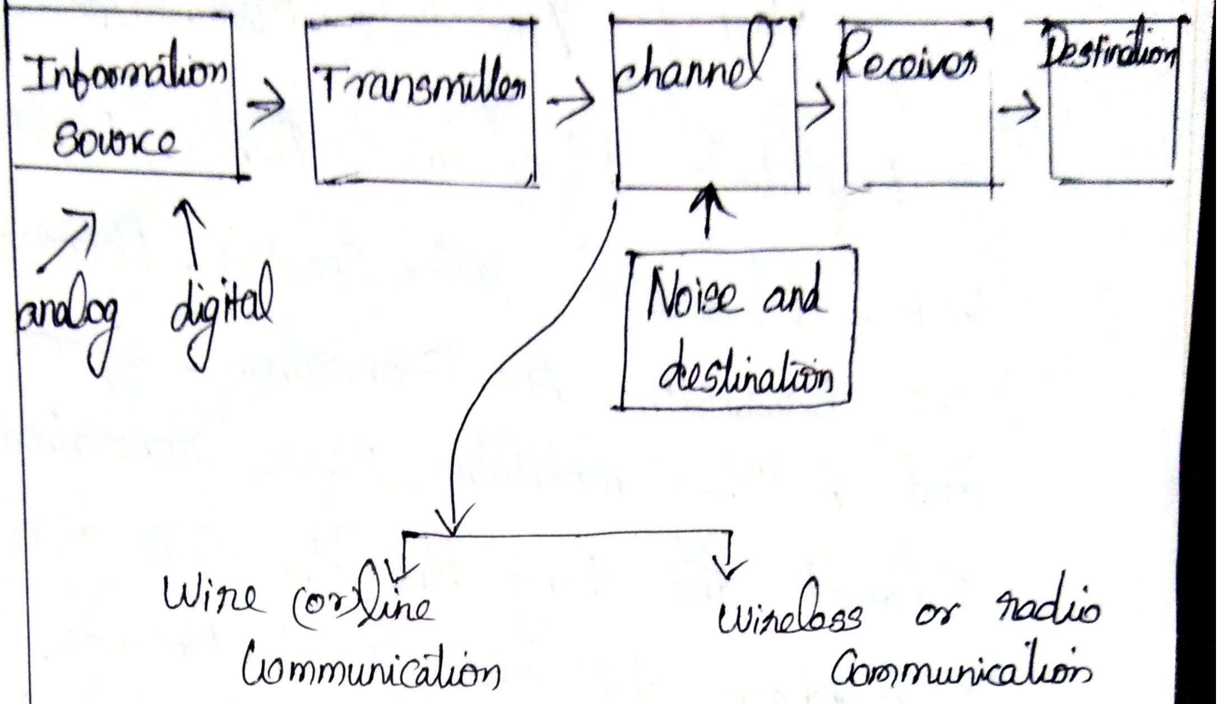
Unit 1 Amplitude Modulation

Amplitude Modulation - DSBSC, DSBFC, SSB. VSB - Modulation index, Spectra, Power relations and Bandwidth - AM Generation - Square law and Switching modulator, DSBSC Generation - Balanced and Ring Modulator. SSB Generation - Filter, phase shift and Third Methods, VSB Generation - Filter method. Hilbert transform, Pre-envelope & Complex envelope - Comparison of different AM techniques, Superheterodyne Receiver.

Introduction

Communication is the process of establishing connection or link between two points for information exchange.

Block diagram of Communication System.



Drawbacks of Baseband Transmission

(i) Large Antenna Height
(ii) Signal get mixed up
(iii) Short range of Communication
(iv) Multiplexing is not possible.
(v) Poor Quality Reception.

The above drawbacks can be overcome by means of modulation techniques.

Modulation

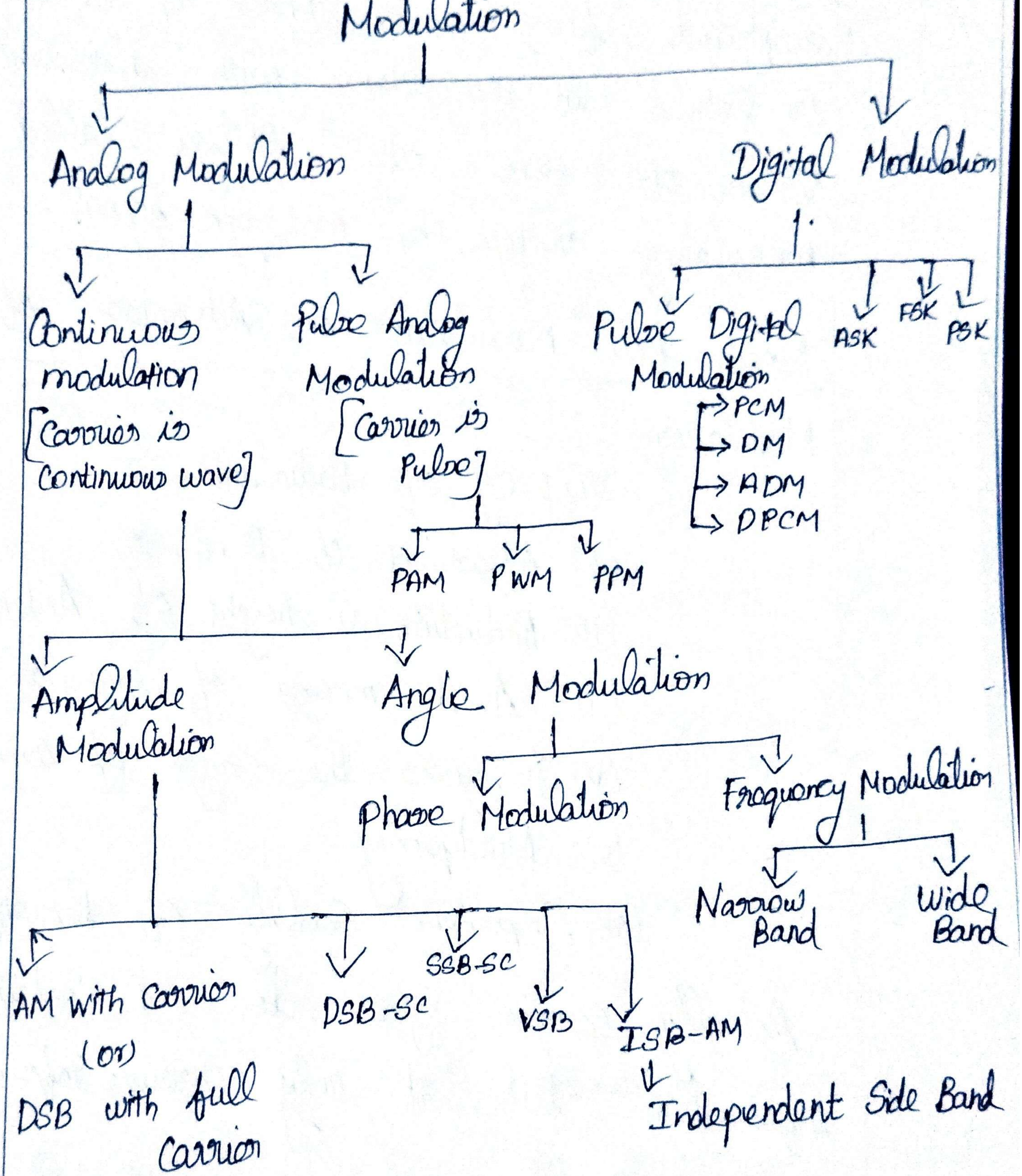
Modulation is defined as the process by which some characteristics, usually amplitude, frequency or phase of carrier wave is varied in accordance with instantaneous value of some other voltage, called modulating voltage (or) message signal.

Need for Modulation (or) Advantage of Modulation

(i) Easy of Radiation
(ii) Adjustment of Bandwidth
(iii) Reduction in height of Antenna
(iv) Avoid mixing of signals
(v) Increases the range of communication
(vi) Multiplexing
(vii) Improves Quality of Reception.

Actually carrier signal does not contain any information but it only carries information

Demodulation is defined as the process of extracting a modulating or baseband signal from the modulated signal.



The amplitude of carrier signal is changed after modulation.

$$V_{AM} = V_c + V_m(t) \rightarrow ③$$

Substitute eq ① in ③

$$V_{AM} = V_c + V_m \sin W_m t$$

$$= V_c\left[1 + \frac{V_m}{V_c} \sin W_m t\right]$$

$$V_{AM} = V_c\left[1 + m_a \sin W_m t\right] \rightarrow ④$$

Hence AM wave is given by

$$V_{AM}(t) = V_{AM} \sin W_c t \rightarrow ⑤$$

Sub ④ in ⑤

$$V_{AM}(t) = V_c\left[1 + m_a \sin W_m t\right] \sin W_c t \rightarrow ⑥$$

(or)

$$V_{AM}(t) = V_c\left[1 + m_a \sin(2\pi f_m t)\right] \sin(2\pi f_c t)$$

$\rightarrow ⑦$

ma → modulation index (or) Depth of modulation

Modulation index

The ratio of maximum amplitude of modulating signal to maximum amplitude

Amplitude Modulation

Amplitude Modulation is the process by Which amplitude of carrier signal is Varied in accordance with instantaneous Value (amplitude) of the modulating signal, but phase and frequency remains Constant.

AM freq range : 450 - 1600 KHz

FM freq range : 80 - 100 Hz.

It is relative inexpensive, low Quality form of modulation that is used for Commercial broadcasting.

Mathematical Representation of AM wave.

Let the modulating signal $V_m(t) = V_m \sin W_m t \rightarrow$ ①

Carrier Signal $V_c(t) = V_c \sin W_c t \rightarrow$ ②

$V_c \rightarrow$ Amplitude of carrier Signal

$V_m \rightarrow$ Amplitude of modulating signal.

$W_m, W_c \rightarrow$ Angular frequency of modulating & Carrier Signal.

Frequency Spectram and Bandwidth

The AM wave is given by

$$V_{AM}(t) = V_c[1 + m_a \sin W_m t] \sin W_c t$$

$$= V_c \sin W_c t + m_a V_c \sin W_m t \sin W_c t$$

$$\left[\sin A \sin B\right] = \frac{\cos(A-B) - \cos(A+B)}{2}$$

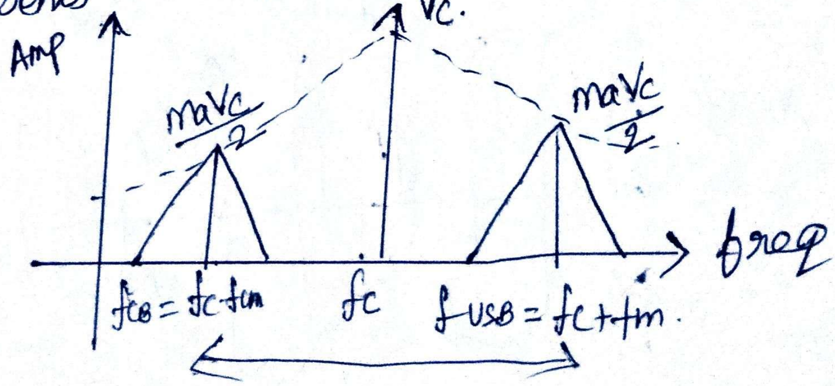
Here $A = W_m$ $B = W_c$

$$V_{AM}(t) = V_c \sin W_c t + \frac{m_a V_c}{2}\left[\cos(W_c - W_m)t - \cos(W_c + W_m)t\right]$$

$$V_{AM}(t) = \underbrace{V_c \sin W_c t}_{\text{Carrier}} + \underbrace{\frac{m_a V_c}{2}\cos(W_c - W_m)t}_{\text{lower Side band}} - \underbrace{\frac{m_a V_c}{2}\cos(W_c + W_m)t}_{\text{Upper Side Band}}$$

Where $\frac{m_a V_c}{2}$ → Amplitude of lower & upper S.B

The negative Sign (-) associated with USB represents the Phase shift of 180°



B.w = 2fm.

of carrier signal.

$$\boxed{m_a = \frac{V_m}{V_c}}$$ $V_m < V_c$ → Avoid distortion.

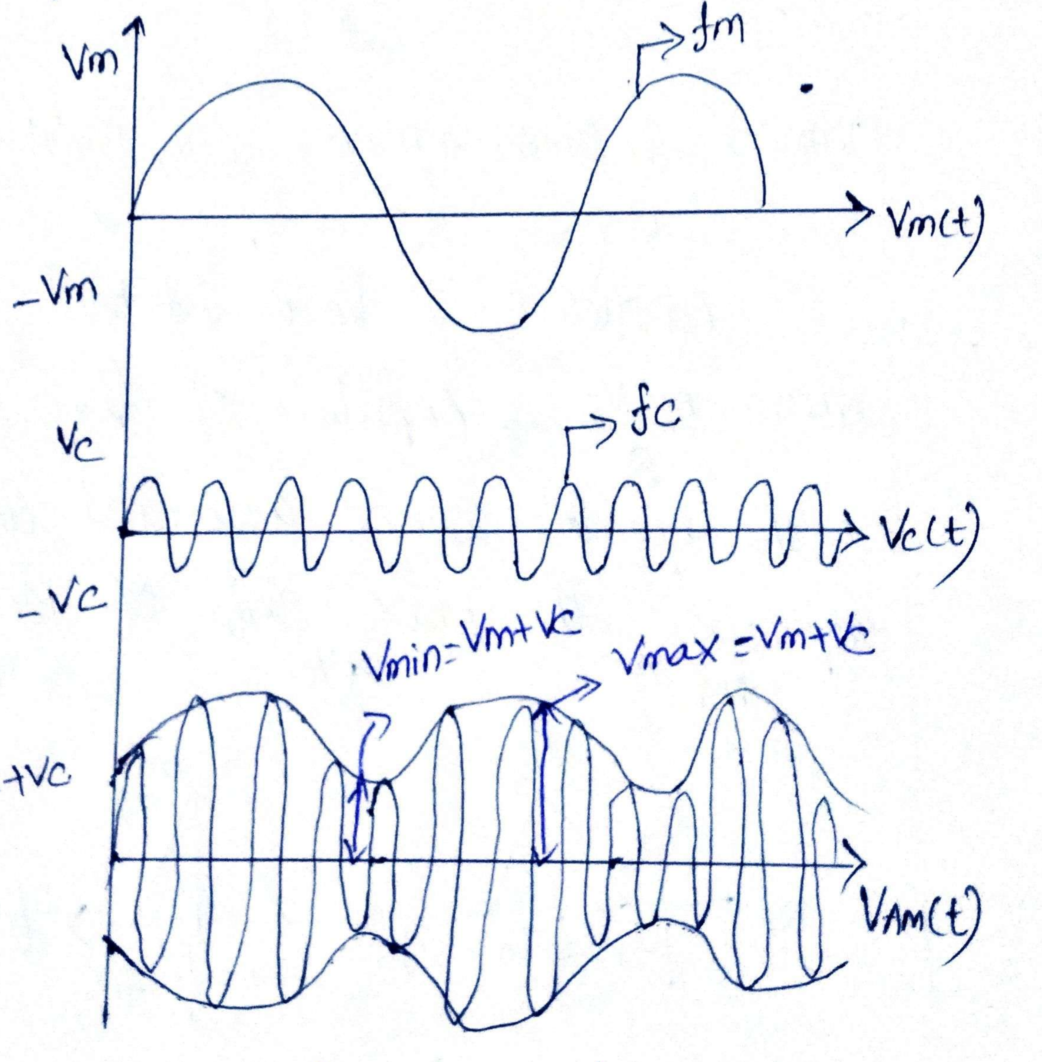
Hence value of ma is between 0 to 1

If $m_a = 1$, then $V_m = V_c$

Percent Modulation

$$\% \text{ Modulation} = m_a \times 100$$
$$= \frac{V_m}{V_c} \times 100$$

for eg $m_a = 0.5$ corresponds to 50% modulation



AM Generation

Power Relations in AM wave.

AM wave consists of carrier and two sidebands. The total power of modulated wave will be

$$P_t = [\text{Carrier power}] + [\text{power in LSB}] + [\text{Power in USB}]$$

$$P_t = P_c + P_{LSB} + P_{USB}.$$

$$= \frac{V_c^2}{R} + \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R}.$$

$$P_c = \frac{(RMS)^2}{\text{Load Resistance}} = \frac{(V_c/\sqrt{2})^2}{R} = \frac{V_c^2}{2R}$$

$(RMS = \text{Amplitude}/\sqrt{2})$

$$P_c = \frac{V_c^2}{2R}$$

III^ly power in sidebands $= P_{LSB} = P_{USB} = \frac{(V_{USB}/\sqrt{2})^2}{R}$

but $V_{USB} = \frac{m_a V_c}{2}$

$$P_{LSB} = P_{USB} = \frac{m_a^2 V_c^2}{8R} = \frac{m_a^2}{4} P_c \qquad \left[\because P_c = \frac{V_c^2}{2R}\right]$$

Total Power:-

$$P_t = P_c + P_{LSB} + P_{USB}$$

$$= \frac{V_c^2}{2R} + \frac{m_a^2}{4}\left(\frac{V_c^2}{2R}\right) + \frac{m_a^2}{4}\left(\frac{V_c^2}{2R}\right).$$

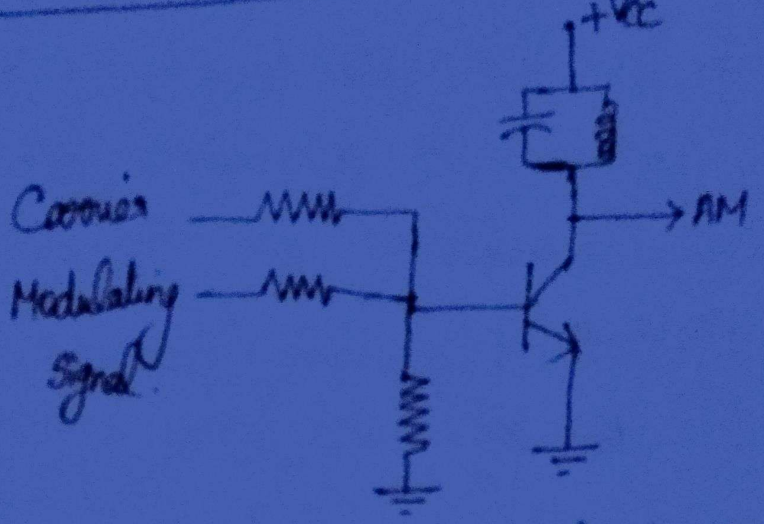
$$P_t = P_c\left[1 + \frac{m_a^2}{2}\right]$$

DSB FC

Double Sideband Full carrier (DSB FC) system contains full carrier and both the side bands.

DSB FC Modulation circuits

Low Level Modulator



The circuit has two input namely RF carrier and modulating signal. When the modulating signal is absent, only the carrier is applied, the circuit works only as a class-A amplifier and get amplified carrier at the output.

When modulating signal is applified. the amplifier operates as non linear device and multiplication of carrier and modulating signa will take place.

The gain of the amplifier is dependent on the modulating signal. The carrier is amplified based on gain variations.

The modulation index ma is proportional to the amplitude of modulating signal. The Voltage gain of emitter modulator is given as

$$A_v = A_a \left[1 + m_a \sin(2\pi f_m t)\right]$$

$\sin(2\pi f_m t)$ goes from maximum value of +1 & minimum value of -1.

$$A_v = A_a [1 \pm m_a].$$

At 100% modulation, $m_a = 1.$ $\Rightarrow A_v(\max) = 2A_a.$

$A_v(\min) = 0.$

<u>operation</u>

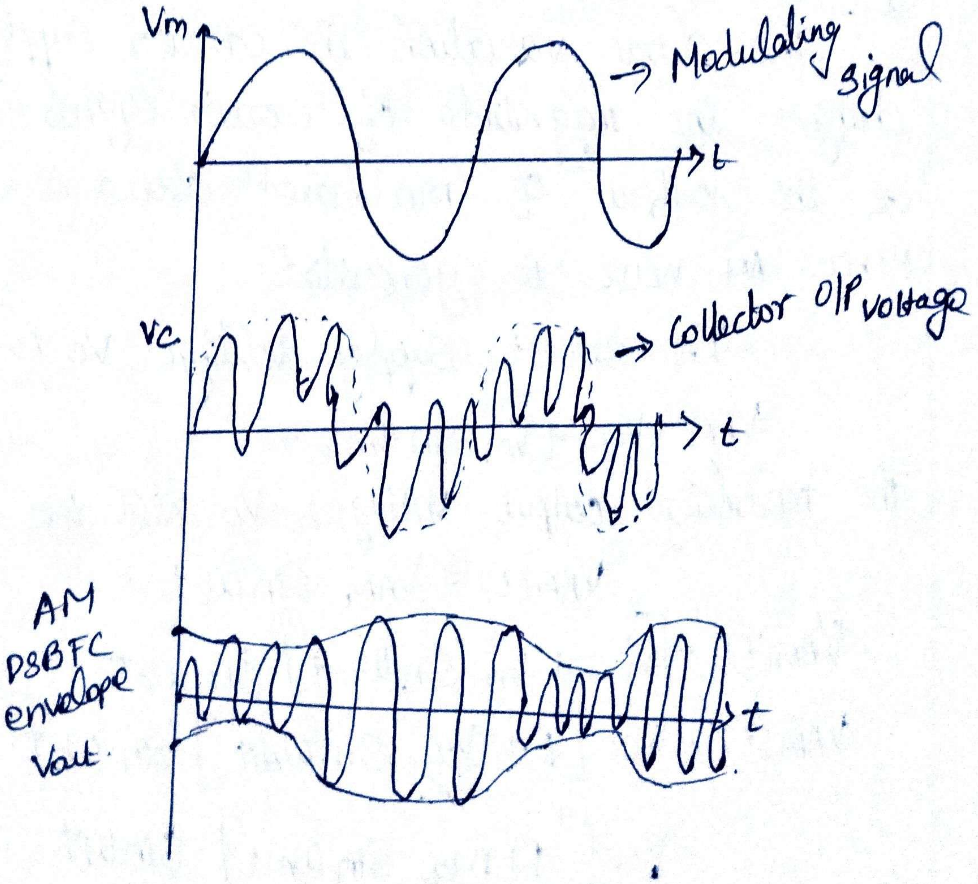
The modulating signal is applied through isolation Transformer. T. to the emitter of Q_1 and carrier is applied to the Base. The modulating signal derives the circuit into both

Saturation and cut off thus producing the non linea amplifiers necessary for modulation to occur.

The collector waveform includes the carrier and upper and lower side frequencies as well as a component at modulating signal frequency.

The unwanted modulating signal from AM waveform is removed by the coupling capacitor C_2, thus producing a symmetrical AM envelope at Vout across R_L.

In emitter modulation, the amplitude of output signal depends on the amplitude of i/p carrier and voltage gain of the amplifier



High Level Modulator [Collector Modulator]

The modulation take place at the Collector terminal i.e. Output stage of transmitter. It has two transistors T_1 & T_2 where T_1 is a high power RF class c amplifier (or) modulated amplifier.

T_2 is a class B amplifier used to amplify the base band signal. Carrier signal is applied to transistor T_2.

When the modulating signal $V_m \sin \omega_m t$ is appears across the modulating Transistor T_1, its voltage will be added with carrier voltage V_{c1}.

The slow variation in carrier supply voltage changes the magnitude of carrier signal voltage at the output of modulated class c amplifier Hence AM wave is generated.

The carrier supply voltage Vc is given by

$$V_{AM} = V_{CC} + V_m \sin \omega_m t.$$

The modulated output voltage Vo will be

$$V_{AM}(t) = V_{AM} \sin \omega_c t$$

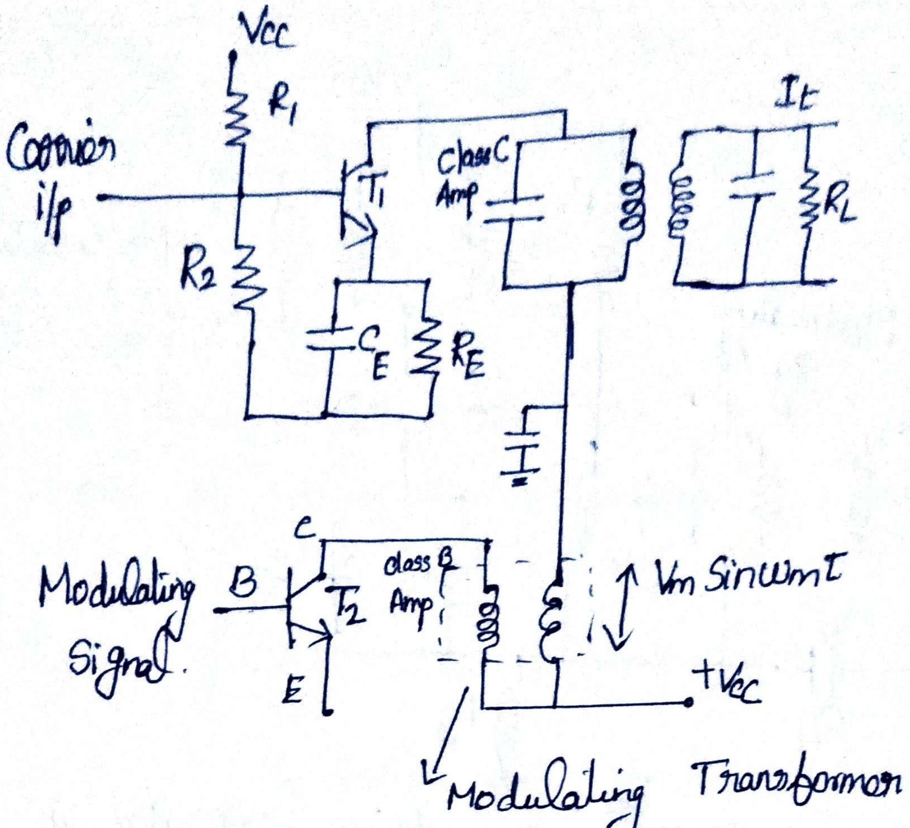
$$V_{AM}(t) = [V_{CC} + V_m \sin \omega_m t] \sin \omega_c t$$

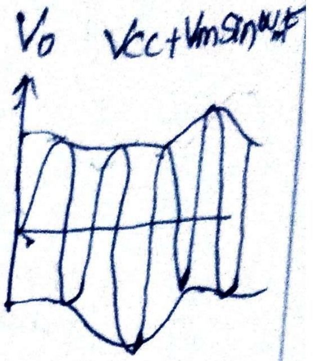
$$V_{AM}(t) = V_{CC}\left[1 + \frac{V_m}{V_C} \sin \omega_m t\right] \sin \omega_c t$$

$$= V_{CC}[1 + m_a \sin \omega_m t] \sin \omega_c t$$

Power efficiency is practically higher than 80%.

Circuit diagram.





Power and efficiency calculation

The modulated power delivered to the output load depends on the input supplied by supply voltage and power dissipation in collector circuit.

Out of total power in collector circuit, only a part of it reaches the output load, the remaining power is lost in collector circuit.

Collector efficiency

$$\eta_c = \frac{P_{out}}{P_{in}}$$

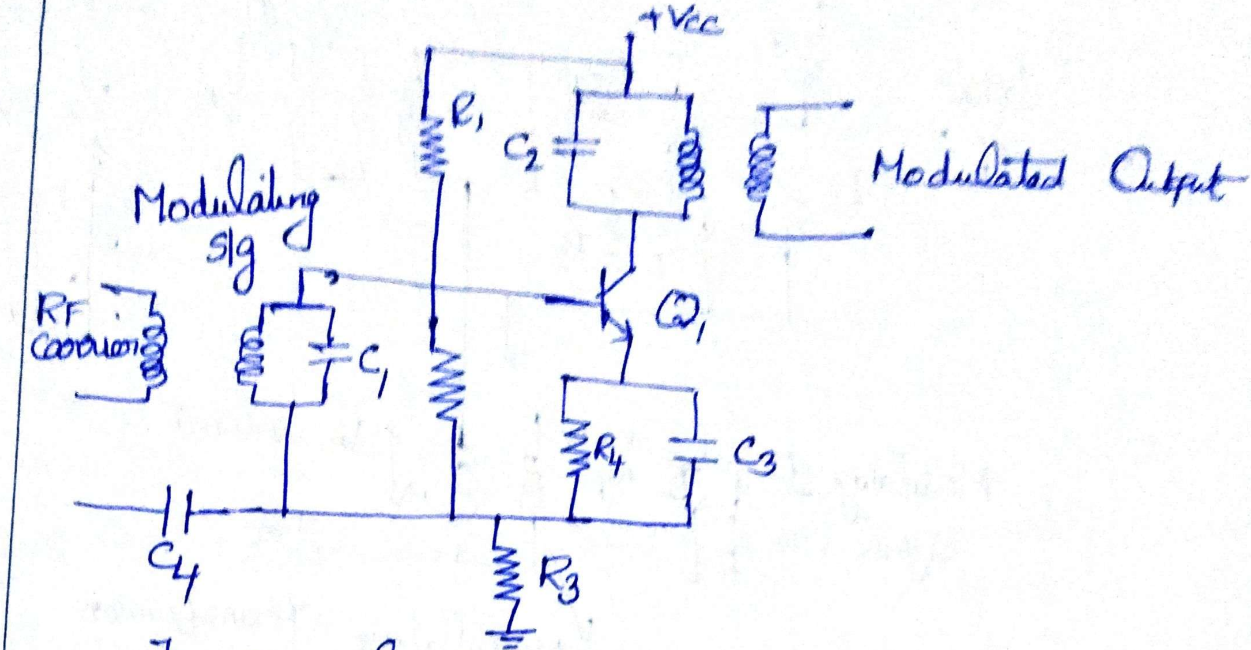
$P_{total} = P_{in} = P_{out} = P_d$.

$$P_{total} = P_{cc}\left(1+\frac{m_a^2}{2}\right)$$

$$P_{in} = P_{cc}\left[1+\frac{m_a^2}{2}\right]$$

$$P_{out} = \eta P_{in} = \eta P_{cc}\left[1+\frac{m_a^2}{2}\right]$$

AM Generation - Square Law Modulator



In general, any device operated in nonlinear region of its output characteristics is capable of producing AM waves when the carrier and modulating signals are fed at the input.

Thus the transmitter, triode tube, a diode etc, may be used as a square law modulator. The above circuit is common emitter configuration. The modulation signal is applied to the emitter and RF carrier at the base of transistor.

A square law modulator circuit consists of

(i) A non linear device.

(ii) A Band pass filter

(iii) A carrier source and modulating signal

The modulating and carrier signal are connected in series with each other.

$$V_1(t) = V_m \sin w_m t + V_c \sin w_c t \rightarrow ①$$

The input output relation for non linear devices as follows.

$$V_2(t) = a V_1(t) + b V_1^2(t) + \dots\dots \rightarrow ②$$

Where a and b are constants

Substitute eq ① in ②, we get

$$V_2(t) = a[V_m \sin w_m t + V_c \sin w_c t] + b[V_m \sin w_m t + V_c \sin w_c t]$$

$$= a[V_m \sin w_m t + a V_c \sin w_c t + b V_m^2 \sin^2 w_m t +$$

$$b V_c^2 \sin^2 w_c t + 2b V_m V_c \sin w_m t + \sin w_c t + \dots\dots$$

① term → modulating signal

② term → Carrier signal

③ term → Square Modulating signal.

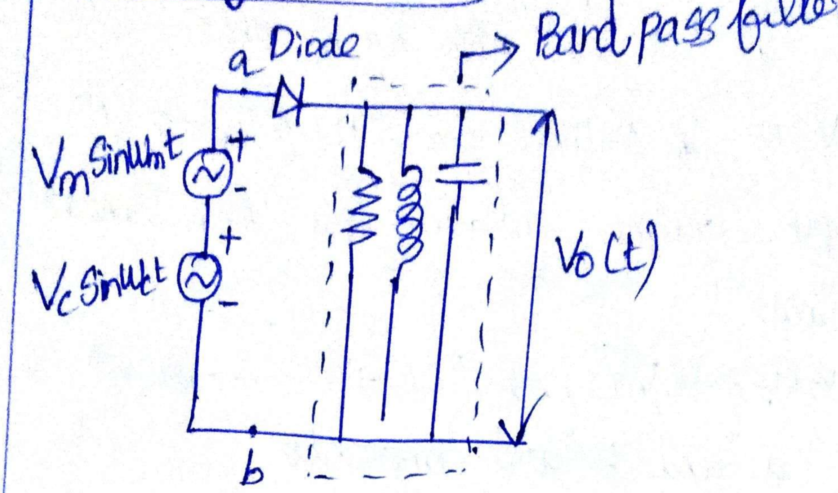
④ term → Squared carrier signal.

⑤ term → AM wave with only sidebands

The LC-tuned circuit acts as a bandpass filter. The circuit is tuned to frequency f_c and its bandwidth is equal to $2f_m$.

$B.w = 2f_m$

Switching Modulator



A Simple diode used for AM Switching Modulator. The diode is forward biased for every positive half cycle of the carrier and behaves like short circuit switch. The signal appears at the input of bandpass filter.

for negative half cycle of the carrier the diode is reverse biased and behave like open switch. The signal does not reach the filter, and no output is obtained. Thus signal is modulated at the rate of carrier frequency.

The Output Voltage is given by

$$V_o(t) = [V_c + V_m \sin\omega_m t] \sin\omega_c t$$

Applications of AM:-

(i) Radio broadcasting

(ii) Picture transmission in a TV system.

Saved due to

The information is contained in two sidebands only. But the sidebands are images of each other and hence both of them contain same information. Transmitting the whole thing cause power wastage and bandwidth also.

Double sideband Suppressed carrier (DSB-SC)

The transmitting wave consists of only the upper and lower sidebands Transmitted power is saved here through suppression of carrier wave. because it does not contain any useful information.

Expression for DSB-SC

Let the Modulating signal, $V_m(t) = V_m \sin W_m t$

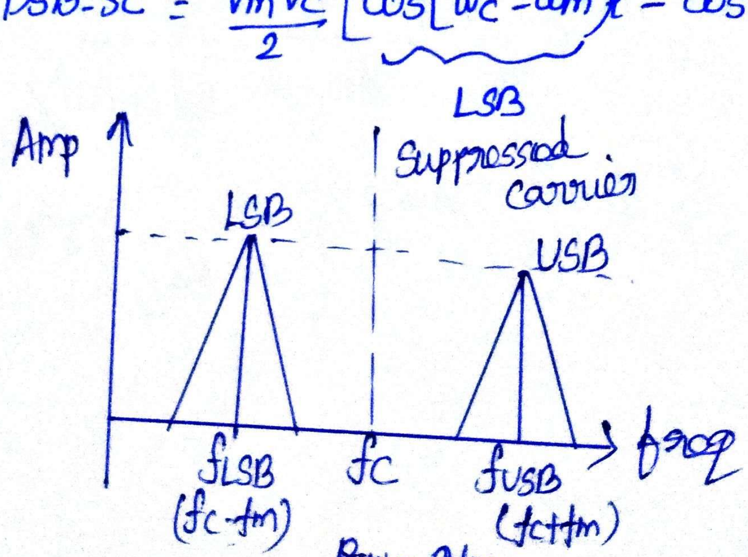
$V_c(t) = V_c \sin W_c t$

$$V(t)_{DSB\text{-}SC} = V_m(t)\, V_c(t)$$

$$= V_m \sin W_m t \; V_c \sin W_c t$$

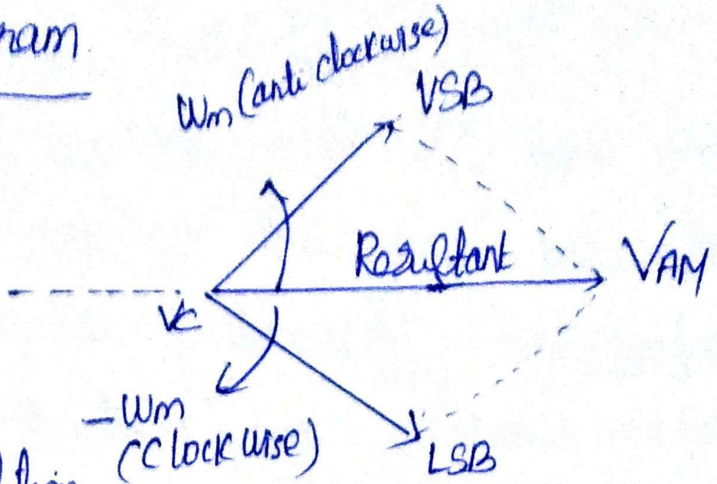
$$= V_m V_c \sin W_m t \sin W_c t$$

$$V(t)_{DSB\text{-}SC} = \frac{V_m V_c}{2}\Big[\underbrace{\cos(w_c - w_m)t}_{LSB} - \underbrace{\cos(w_c + w_m)t}_{USB}\Big]$$



Bw = 2fm

Phasor Diagram



Power Calculation

Total power transmitted in AM is

$$P_t = P_{carrier} + P_{LSB} + P_{USB}$$

$$= \frac{V_c^2}{2R} + \frac{m_a^2 V_c^2}{8R} + \frac{m_a^2 V_c^2}{8R} = \frac{V_c^2}{2R} + \frac{m_a^2 V_c^2}{4R}.$$

$$P_t = \frac{V_c^2}{2R}\left[1 + \frac{m_a^2}{2}\right]$$

$$P_t = P_c\left[1 + \frac{m_a^2}{2}\right] \quad \text{where } P_c = \frac{V_c^2}{2R}.$$

If the carrier is suppressed, then the total power transmitted in DSB-SC-AM is

$$P_t' = P_{LSB} + P_{USB}$$

$$P_t' = \frac{m_a^2 V_c^2}{8R} + \frac{m_a^2 V_c^2}{8R} = P_c \frac{m_a^2}{2}$$

$$\text{Power Savings} = \frac{P_t - P_t'}{P_t} = \frac{1}{1 + m_a^2/2}$$

$$\text{\% power Saving} = \left(\frac{1}{1 + m_a^2/2}\right) \times 100 = 66.67\%$$

$$\underline{(m_a = 1)}$$

In DSB-SC, 66.7% of power is saved due to the suppression of carrier wave.

Generation of DSB-SC-AM

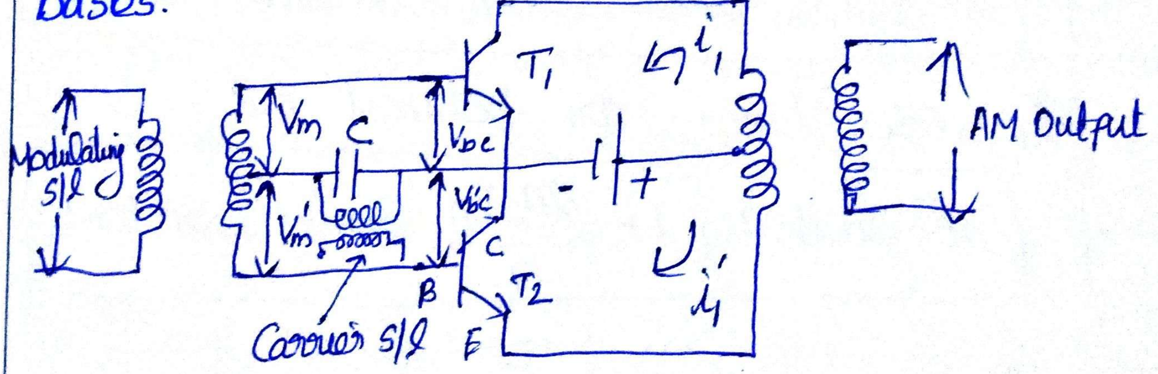
Balanced Modulator

Ring Modulator
(or)
Diode Balanced Modulator
(or)
Double Balanced Modulator.

Balanced Modulator

The Modulating Voltage across the two windings of a centre tap transformer are equal and opposite in phase. i.e., $V_m = -V_m'$

The carrier is applied to the centre tap of i/p transformer and is in phase at base of T_1 & T_2. The modulated signal is antiphase at the two bases.



The input Voltage to transistor T_1 is given by

$$V_{bc} = V_c + V_m$$
$$= V_c \sin W_c t + V_m \sin W_m t$$

Similarly, the input voltage to transistor T_2 is given by

$$V_{bc}' = V_m' + V_c.$$

$$= -V_m \sin W_m t + V_c \sin W_c t$$

By non Linearily relationship,

$$i_1 = a_1 V_{bc} + a_2 V_{bc}^2$$

$$i_1' = a_1 V_{bc}' + a_2 V_{bc}^2$$

Substitute the values of V_{bc} and V_{bc}'

$$i_1 = a_1 [V_c \sin W_c t + V_m \sin W_m t] + a_2 [V_c \sin W_c t + V_m \sin W_m t]^2$$

$$i_1' = a_1 [V_c \sin W_c t - V_m \sin W_m t] + a_2 [V_c \sin W_c t - V_m \sin W_m t]^2$$

The output AM Voltage V_0 is given by

$$V_0 = k(i_1 - i_1')$$

This is because current i_1 & i_1' flow in opposite direction in a tuned circuit.

$$\boxed{V_0 = 2ka_1 V_m \sin W_m t + 4ka_2 V_c V_m \sin W_c t \sin W_m t}$$

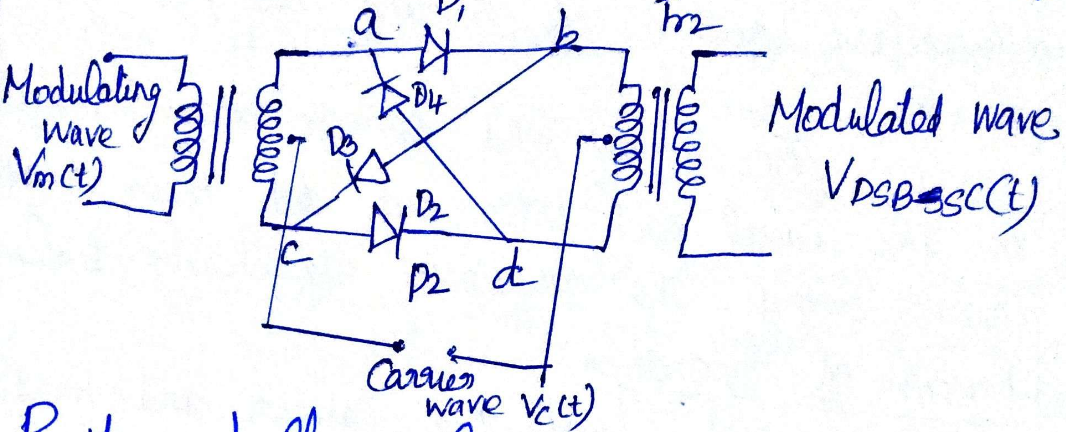
The other terms are balanced out.

$$\boxed{V_0 = 2k V_m a_1 \left[1 + \frac{2a_2 V_m}{a_1} \sin W_c t\right] \sin W_m t}$$

where $m_a = \frac{2a_2 V_m}{a_1}$ is modulation index.

Ring Modulator (or) Diode Balanced Modulator.

It is one of the most popular method of generating a DSB-SC wave. The circuit employs diodes as non linear devices and the carrier signal is connected between centre taps of the input and output transformers. The four diodes are controlled by a carrier $V_c(t)$ of frequency f_c.



Positive half cycle of carrier :-

Diodes D_1 and D_2 are forward biased. At the time D_3 & D_4 are reverse biased and acts like open circuits. The current divices equally in the upper and lower portions of the primary windings of Tr2.

The current in upper part of the winding produces a magnetic field that is equal and opposite to the magnetic field produced by the current in lower half of the secondary.

These magnetic fields cancel each other out and no output is induced in the secondary. Thus the carrier is effectively suppressed.

Negative half cycle of carrier

When the polarity of the carrier reverses Diodes D_1 and D_2 are reverse biased and diodes D_3 and D_4 conduct. Again the current flows in the secondary winding of Tr_1 and the primary windings of Tr_2.

The equal and opposite magnetic fields produced in Tr_2 cancel each other out and thus result in zero carrier output. The carrier is effectively balanced out.

Principle of operation

When both the carrier and modulating signal are present, during positive half cycle of the carrier, diodes D_1 and D_2 conduct, while diodes D_3 and D_4 does not conduct.

During negative half cycle of the carrier voltage diodes D_3 and D_4 conduct and D_1 & D_2 does not conduct.

Phase reversal.

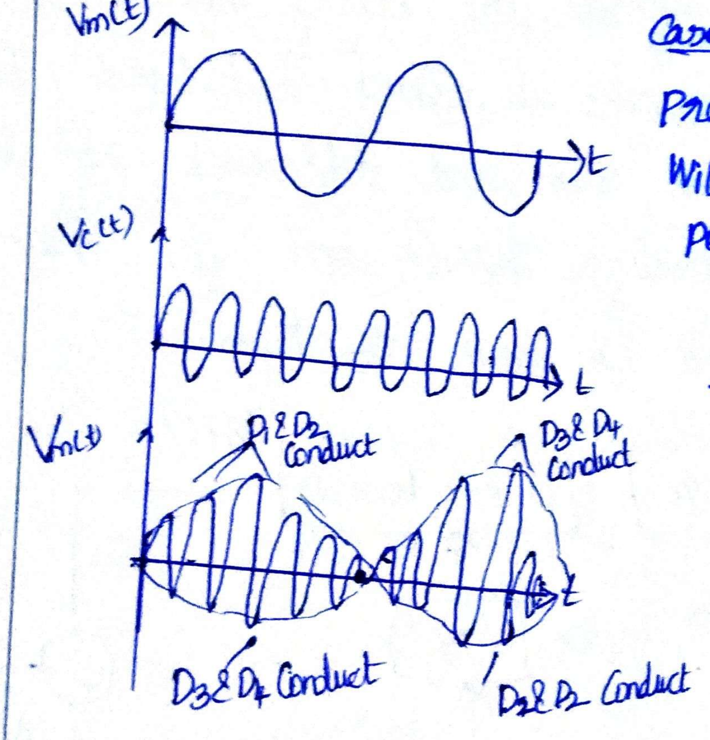
When polarity of modulating signal changes, the result is a 180° phase reversal.

At the time, during the positive half cycle of the carrier, diode D_3 and D_4 are is forward bias and negative half cycle of the carrier, diodes D_1 & D_2 are in reverse bias.

$$V_O(t) = V_m(t)\, V_c(t)$$

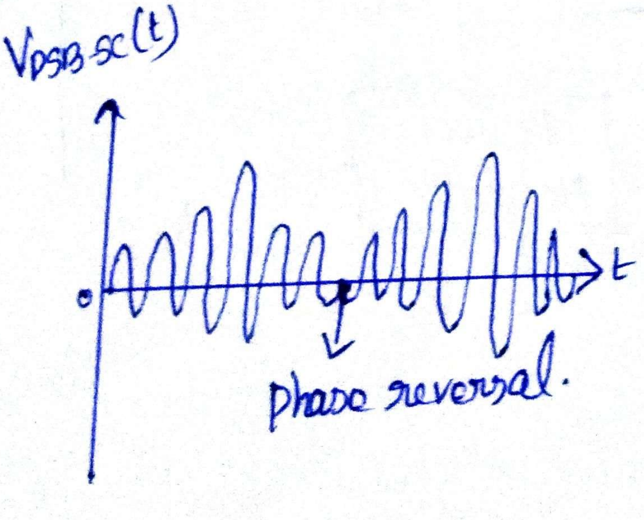
$$= \frac{V_m V_c}{2}\left[\underbrace{\cos(\omega_c - \omega_m)t}_{LSB} - \underbrace{\cos(\omega_c + \omega_m)t}_{USB}\right]$$

The ring modulator circuit is also known as double balanced modulator. Because comparing to balanced modulator, here two more diodes are used.



Case (i) When Modulating signal Present diodes D_1, D_2 or D_3, D_4 will conduct depends on signal polarity.

Case (ii) When Carrier signal alone present, the flow of current in two halves of output transformer is equal & opposite. and no output can develop across the load.



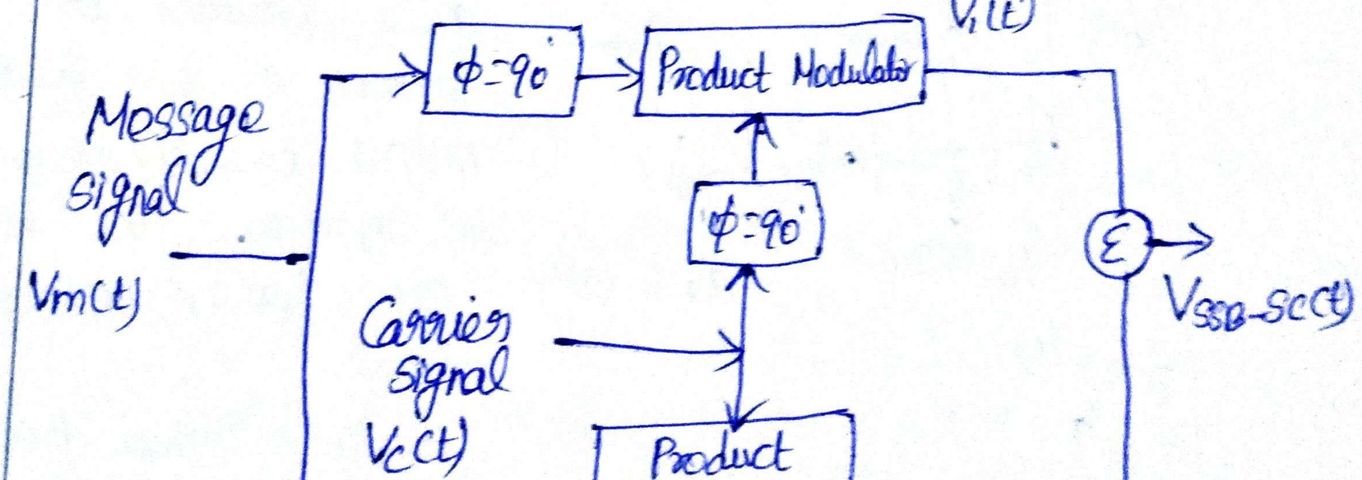
Case (iii) When both signals are present, the resultant potential is one half of the output transformer becomes larger than other.

It is more efficient in transmitted power and better signal to noise ratio Compared to DSBFC&SSB transmission.

Eventhough Carrier is suppressed the Bandwidth of DSBFC remains same as DSBFC. The output is free from carrier and contains upper and lower sidebands only.

Single Side band Suppressed Carrier [SSB-SC]

In DSB signal, the basic information is transmitted twice, once in each side band. The Sidebands are the sum and difference of the Carrier and modulating signals and the information must be contained in both of them.



So, either one sideband is enough for transmitting as well as recovering the useful message. One side band may be suppressed. The remaining sideband is called a single sideband carrier (SSB-SC) signal.

SSB requires half of the bandwidth of the DSB-SC use considerably less transmitted power.

$\boxed{B.W = f_m.}$ [∵ B.W of AM=2fm]

In order to suppress one of the sidebands, the input signal fed to the modulator 1 is 90° out of phase with that of the signal fed to the modulator 2.

let $V_1(t) = V_m \sin(\omega_m t+90)\ V_c \sin(\omega_c t+90)$

$V_1(t) = V_m \cos \omega_m t + V_c \cos \omega_c t$

$V_2(t) = V_m \sin \omega_m t + V_c \sin \omega_c t$:

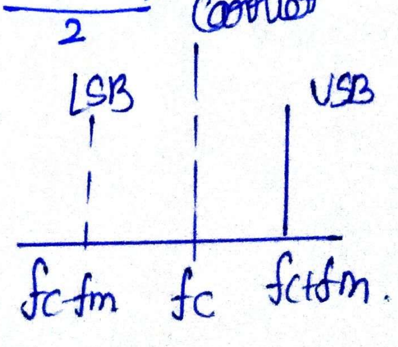
$V_{SSB}(t) = V_1(t) + V_2(t)$

$= V_m V_c [\sin \omega_m t \sin \omega_c t + \cos \omega_m t \cos \omega_c t]$

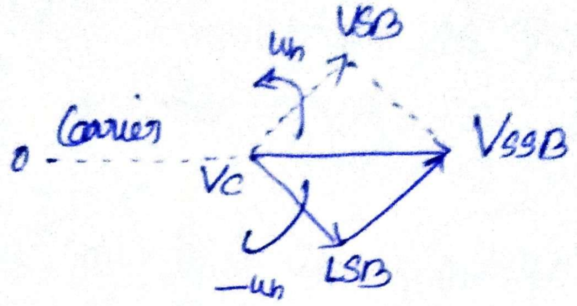
We Know that

$\sin A \sin B + \cos A \cos B = \frac{\cos(A-B)}{2}$

$V_{SSB}(t) = \frac{V_m V_c}{2} \cos(\omega_c - \omega_m)t$



Phasor diagram.



Power Calculation

Power in SSB-SC AM is

$$P_t'' = USB (or) LSB = \frac{1}{4} m_a^2 P_c.$$

Power Savings with respect to AM with Carrier

$$= \frac{P_t - P_t''}{P_t}$$

$$= \frac{\left[1 + \frac{m_a^2}{2}\right] P_c - \left[\frac{m_a^2}{4} P_c\right]}{\left[1 + \frac{m_a^2}{2}\right] P_c}$$

$$\text{Power Saving} = \frac{1 + \frac{m_a^2}{2} - \frac{m_a^2}{4}}{1 + \frac{m_a^2}{2}} = \frac{4 + m_a^2}{4 + 2m_a^2}$$

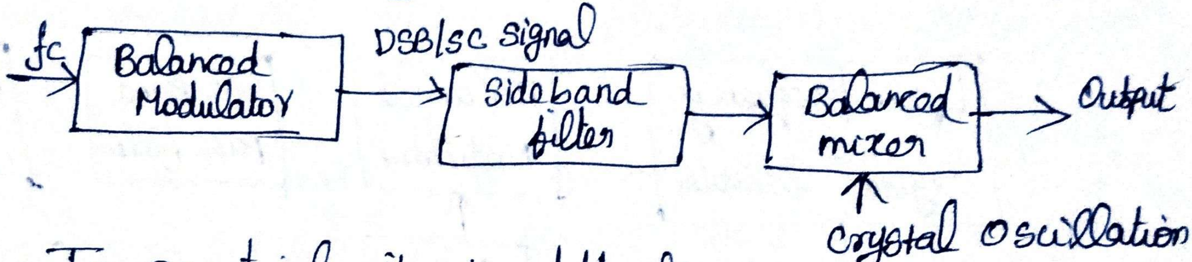
If $m_a = 1$, then $\eta = 83.33\%$. In addition to Carrier, one of the sidebands also suppressed.

Generation of SSB

- Frequency Discrimination (or) Filter Method (or) Balanced Modulator
- Phase Discrimination
  - Phase shift Method (or) Phasing Method
  - Weavers Method (or) Modified phase shift

Filter Method

The method basically consists of a balanced modulator (to generate DSB sc signal) and Suppression filter [to remove unwanted sidebands]



In practical, it is difficult to design a filter with a sharp cut off frequency on other side. If the Bandwidth is reduced in an effort to eliminate the unwanted side band, such a filter will introduce attenuation in the unwanted sideband also. Increasing the Bandwidth may result in passing some of the unwanted sidebands to the output.

The filter must have flat pass band and extremely high attenuation outside the pass band. Hence Q factor of this type of tuned circuit must be very high. The value of Q factor increases as the difference between modulating carrier frequency increases.

Q factor can be expressed as

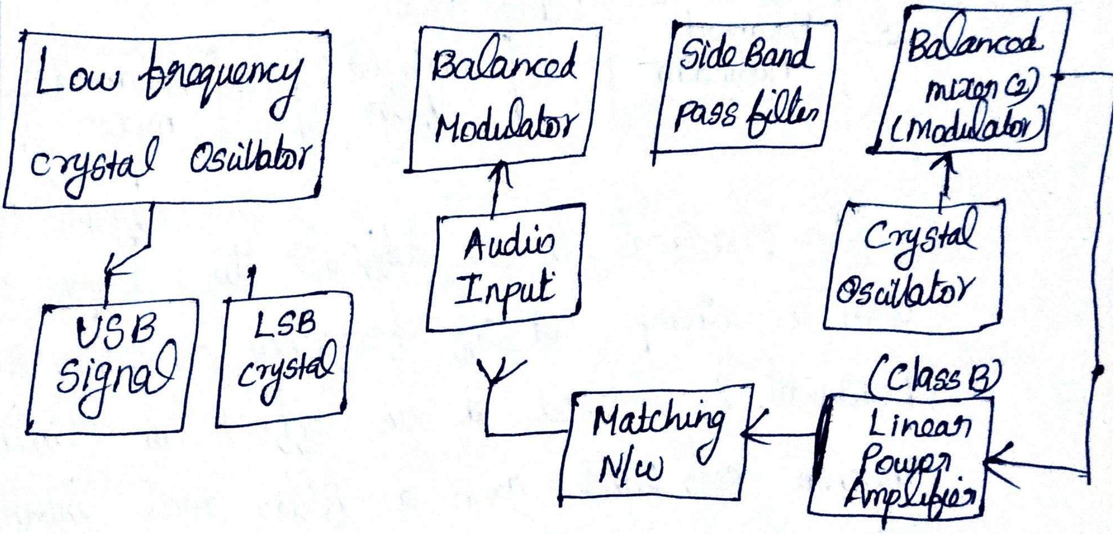
$$Q = \frac{fc\left[\log^{-1} S/20\right]^{1/2}}{4\Delta f}$$

Q-Quality factor

fc → Carrier frequency

S → dB level of suppression of unwanted Sideband

Δf ⇒ frequency separation between Sidebands.



The filtered signal is upconverted in second balanced modulator (mixer) to the final transmitter frequency. and then amplified before being coupled to the antenna.

Linear power amplifier are used to avoid distortion & the sideband signal. class B is more efficient that class A

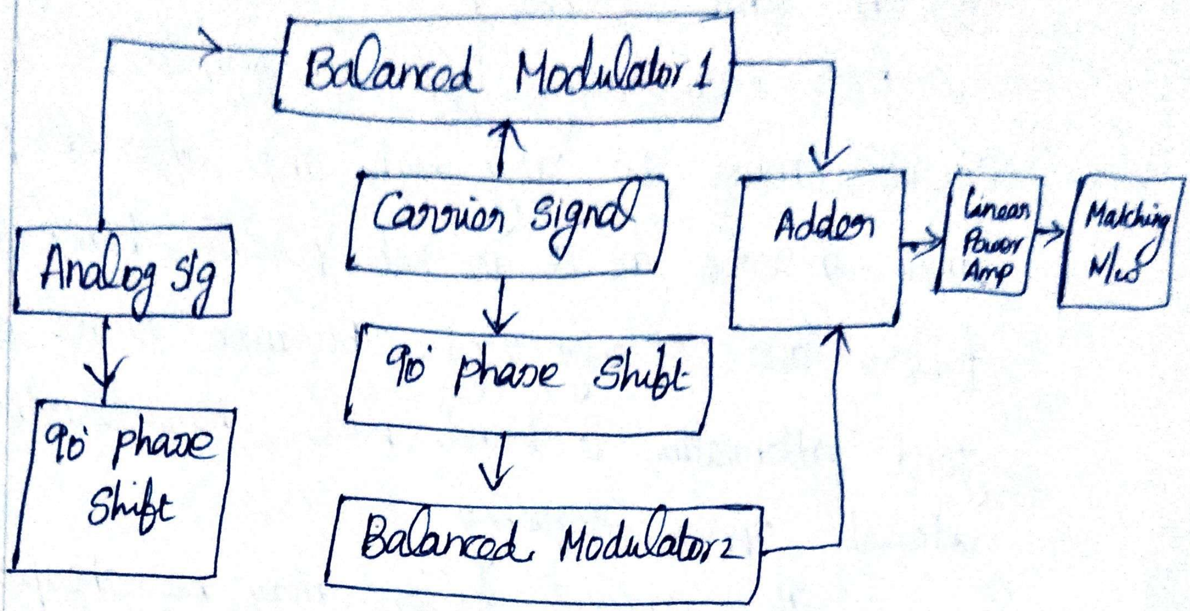
For transmitting high frequencies, Q of tuned circuits must be very high and after a particular limit increase in Q is not possible. hence Mechanical filters are oftenly used. Because small size, very good attenuation & band pass characteristics, and adequate upper frequency.

The crystal filters may be cheaper but are preferable only at frequencies greater than 1 MHz.

The balanced ~~to filter~~ mixer is similar to balanced modulator except that its sum frequency is away from the crystal oscillator frequency.

It is difficult to filter out the unwanted frequencies in the output of the mixer if transmitting frequency is much higher than operating frequency.

Phase Shift Method



The modulating signal and carrier signals are fed into balanced modulator 1 in the usual manner. The balanced modulator 2 is given these signals after a phase shift of 90°

The unwanted sidebands filters can be removed by generating the components of sidebands out of phase. The undesired sideband is USB and then two USB's are generated such that they are 180° out of phase with each other. So that USB's add with each other and cancel out each other.

Two balanced modulators and two phase shifters are used in this phasing method. The carrier signal is cancelled out in this circuit by both of the balanced modulator and unwanted sidebands cancel at the output of summing amplifiers.

I/P sig for modulator 1

$$V_c(t) = V_c \sin \omega_c t \rightarrow ①, \quad V_m(t) = V_m \sin \omega_m t \rightarrow ②$$

I/P signal for modulator 2

$$V_c(t) = V_c \sin(\omega_c + \pi/2)t = V_c \cos \omega_c t \rightarrow ③$$

$$V_m(t) = V_m \sin(\omega_m + \pi/2)t = V_m \cos \omega_m t \rightarrow ④$$

Output for modulator 1

$$V_1(t) = V_c \sin \omega_c t \, V_m \sin \omega_m t$$

$$= \frac{1}{2} V_m V_c \left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$$

Output from Modulator 2

$$V_2(t) = V_m \cos \omega_c t \cos \omega_m t$$

$$= \frac{1}{2} V_m V_c \left[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t\right]$$

The output from linear summer is $V_1(t) + V_2(t)$

$$V_1(t) + V_2(t) = V_m V_c \cos(\omega_c - \omega_m)t$$

The output of two balanced modulator are summed to produce lower side band signal. Thus one of the sideband is cancelled, whereas other is reinforced.

Advantages

It provides the easy of switching from one sideband to the other.

It does not require any sharp cut off filter.

It has ability to generate SSB at any frequency.

Dis Advantages

Since we are using two balanced modulators, each should have equal sensitivity and give exact same output.

The carrier phase shift network must provide an exact 90° phase shift at carrier frequency.

Modified Phase shift Method (or) Third Method

This method is overcome the limitation of phasing method.

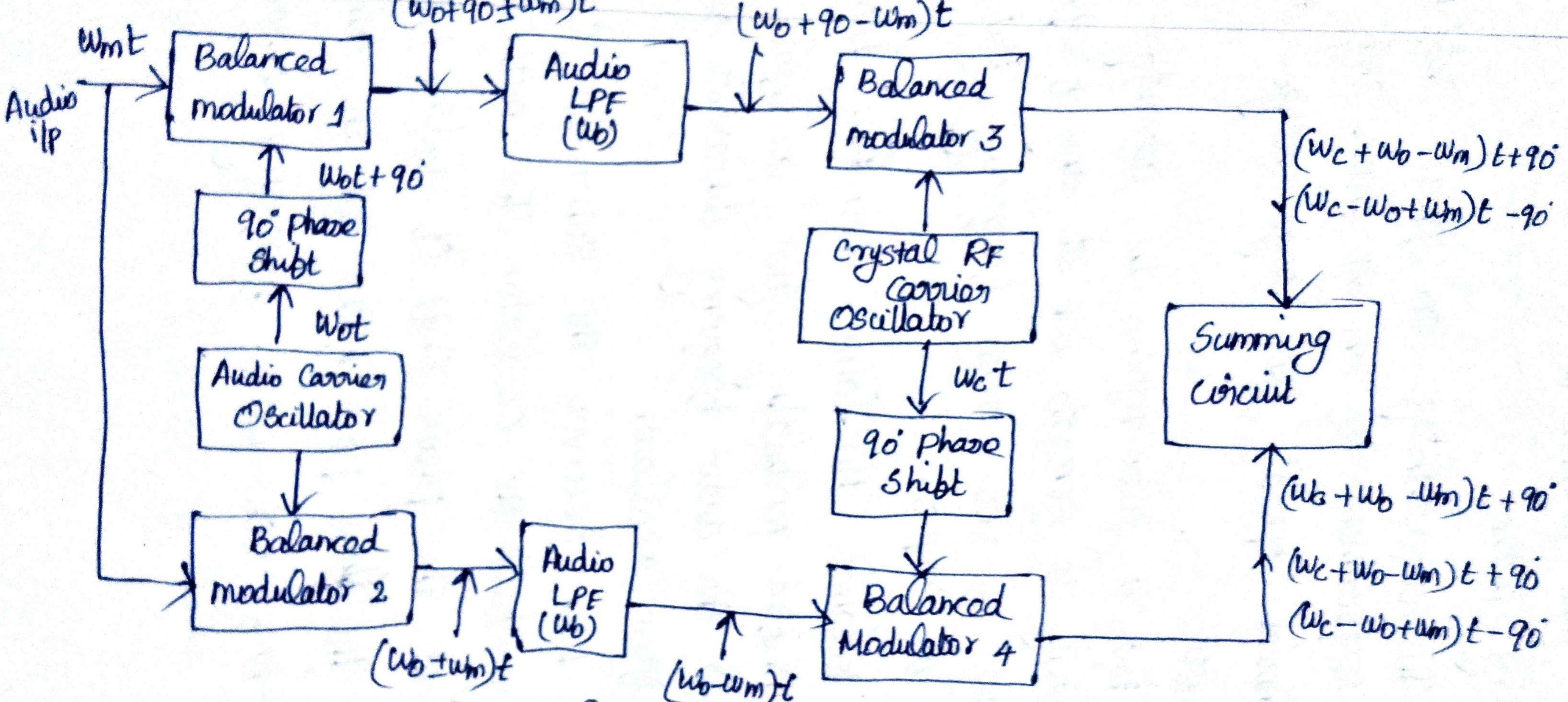
The disadvantage of phase shift method is the requirement of an AF phase shift circuit which should operate over large range audio frequencies.

But it also retains the advantage like its ability to generate SSB at any frequency and use of low audio frequency. But the circuit is complex.

Balanced Modulators BM_1, BM_2 both have the unshifted modulating signal as inputs. Once BM take the audio frequency subcarrier with a 90° shift from oscillator

BM_2 receives the subcarrier signal directly from the oscillator. This method tries to avoid the phase shift of audio frequencies

The low pass filter at the output of BM_1 & BM_2 with cutoff frequency ensures the input to the BM3 & BM4. The output of BM3 and BM4 gives the desired sideband suppress



<u>Output of SSB signal</u>

The modulating signal $V_m(t) = V_m \sin \omega_m t$

AF carrier S/g $V_o(t) = 2 V_o \sin \omega_b t$

RF carrier S/g $V_c(t) = 2 V_c \sin \omega_c t$

Output of Balanced modulator 1

$$= 2V_0 \sin(\omega_0 t + 90)\, V_m \sin\omega_m t$$

$$= V_m V_0 \left[\cos(\omega_0 t - \omega_m t) + 90) - \cos(\omega_0 t + \omega_m t) + 90\right]$$

Output of Balanced Modulator 2

$$= 2V_0 \sin\omega_0 t\, V_m \sin\omega_m t$$

$$= V_m V_0 \left[\cos(\omega_0 t - \omega_m t) - \cos(\omega_0 t + \omega_m t)\right]$$

The low pass filters in the BM_1 & BM_2 eliminates the upper sidebands of modulator

Output of LPF_1 is $V_m V_0 \cos(\omega_0 t - \omega_m t) + 90$

Output of LPF_2 is $V_m V_0 \cos(\omega_0 - \omega_m)t$

Assume $V_m = V_0 = V_c = 1$

Output of Balanced Modulator 3

$$= 2 \sin\omega_c t + \cos(\omega_0 t - \omega_m t + 90)$$

It is in the form of $\sin A \sin B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$$\therefore \sin\left[[\omega_c + \omega_0 - \omega_m]t + 90\right] + \sin\left[(\omega_c - \omega_0 + \omega_m)t + 90\right]$$

Output of Balanced Modulator 4

$$= 2 \sin(\omega_c t + 90) \cos(\omega_0 - \omega_m)t$$

$$= \sin((\omega_c + \omega_0 - \omega_m)t + 90) + \sin\left[(\omega_c - \omega_0 + \omega_m)t + 90\right]$$

From eq ⑦ & ⑧ . the output of summer circuit is

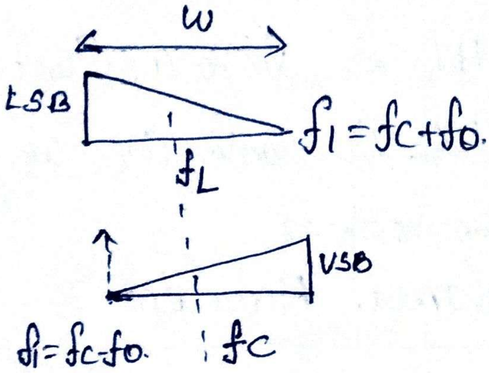
$$V_o = \sin\left[(\omega_c + \omega_o - \omega_m)t + 90\right] + \sin\left[[\omega_c - \omega_o + \omega_m]t - 90\right] +$$

$$\sin\left[(\omega_c + \omega_o - \omega_m)t + 90\right] + \sin\left[(\omega_c - \omega_o + \omega_m)t + 90\right]$$

$$= 2\sin\left[(\omega_c + \omega_o - \omega_m)t + 90\right]$$

$$\boxed{V_o = 2\cos(\omega_c + \omega_o - \omega_m)t}$$

To find RF output frequency is $f_c + f_o - f_m$ which is essentially the lower sideband of RF carrier $f_c + f_o$



Advantage of SSB

(i) Since only single sideband is transmitted. the BW of transmitter and channel is only f_m.

(ii) power of suppressed carrier and sideband is saved.

(iii) Because of narrow Bandwidth of SSB, the effect of noise at the receiver circuits is reduced. This gives better quality of reception in SSB

Application

(i) point to point radio telephone communication

(ii) SSB Telegraph system.

(iii) police wireless communication

(iv) VHF and UHF Communication.

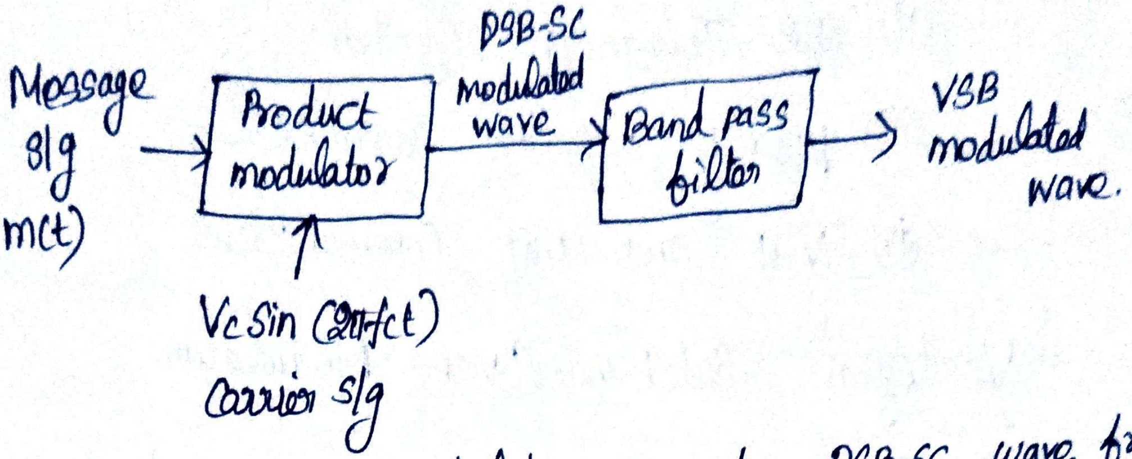
Vestigial Sideband (VSB) Modulation

SSB-SC signals are relatively difficult to generate due to difficulty in isolating desired Sideband. The required filter must have a very sharp cut off characteristics, particularly when the baseband signal contains extremely low frequencies. [eg: Television & Telegraphic Signals]

This difficulty is overcome by a scheme Known as ~~way~~ vestigial sideband modulation which is a Compromise between SSB-SC and DSB-SC modulation

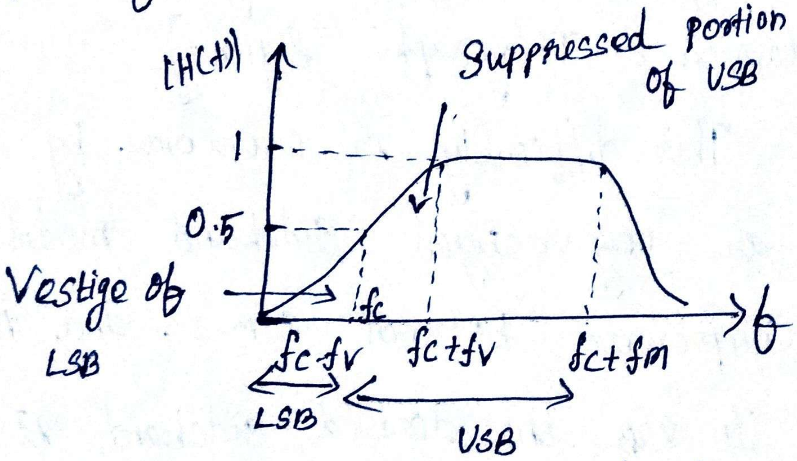
In VSB, the desired sideband is allowed to pass completely. Whereas just a small portion (called trace vestige) of the undesired sideband is allowed. The transmitted vestige of undesired sideband compensates for the loss of the wanted sideband.

Generation of VSB - Filter method



The product modulator generates DSB-SC wave from message signal and carrier signal.

This DSB-SC signal is given to input of BPF, which reject or suppress any one sideband and passes a portion of other sideband.



The portion from fc to fc+fm in USB. The portion from fc to fc+fv is suppressed partially.

The portion from f_c to $f_c - f_m$ in LSB. Its portion from $f_c - f_v$ to f_c is to be transmitted as vestige.

The filter response is only for positive frequencies. The frequency response is normalized, So that carrier frequency $|H(f_c)| = \frac{1}{2}$.

If the transition interval $f_c - f_v \le |f| \le f_c + f_v$, the following two conditions are satisfied.

(i) Sum of values of magnitude response $|H(f)|$ at any two frequencies equally displaced above & below f_c is unity.

(ii) The phase response $[\arg H(f)]$ is unity.

$H(f)$ Satisfies the Condition.

$$H(f-f_c) + H(f+f_c) = 1 \quad \text{for} \quad -f_m \le f \le f_m.$$

Transmission Bandwidth

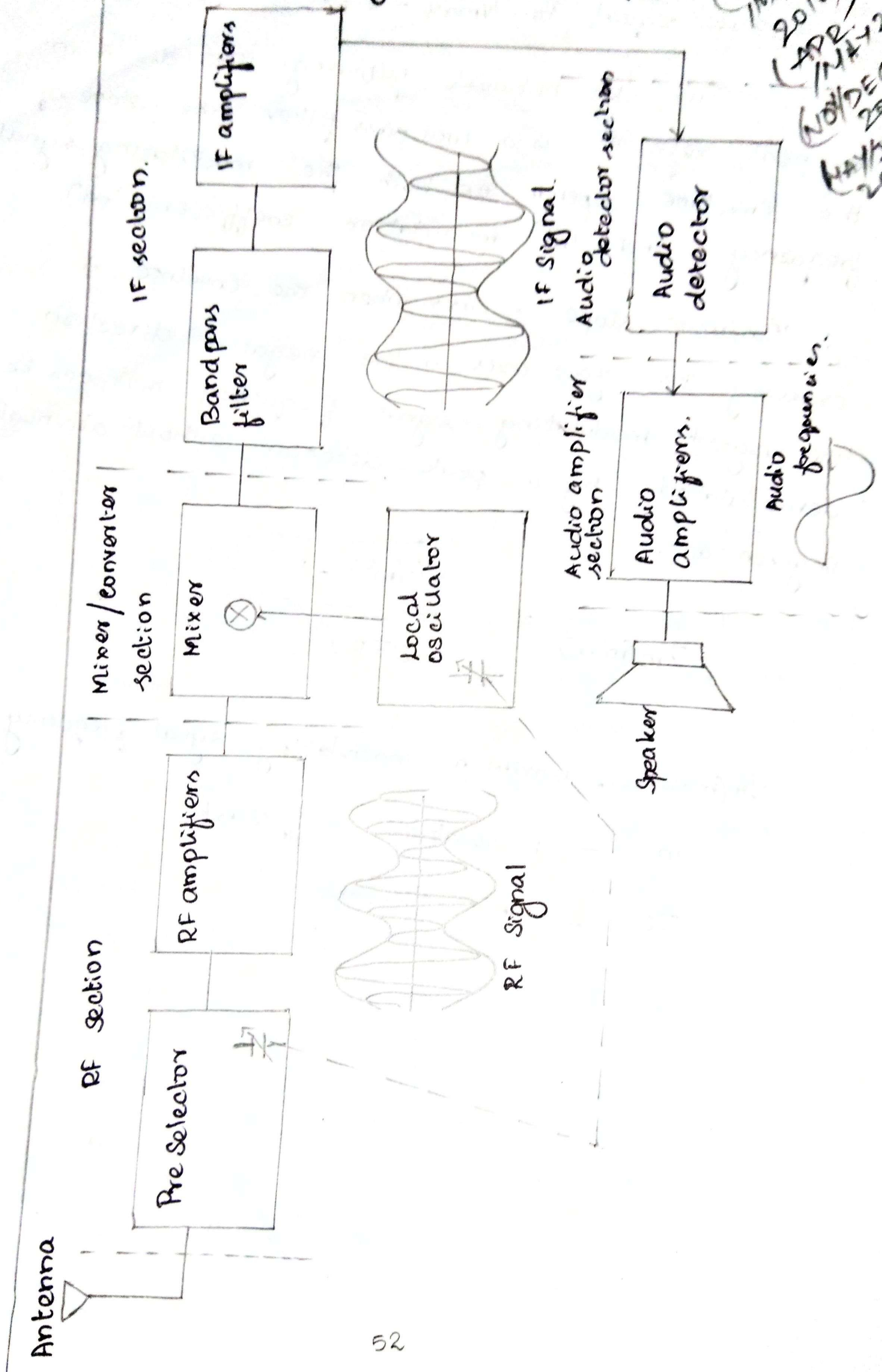
$$BW = \underbrace{f_m}_{\text{message B.w.}} + \underbrace{f_v}_{\text{width of VSB}}$$

VSB Modulated wave in time domain is

$$S(t) = \frac{1}{2} V_c\, m(t) \cos(2\pi f_c t) \pm \frac{1}{2} V_c\, \hat{m}(t) \sin(2\pi f_c t)$$

Super Heterodyne Receiver :: (AM)

(APR/MAY 2018) (APR/MAY 2017) (NOV/DEC 2015) (MAY/JUNE 2016)



Heterodyne means to mix two frequencies together in a nonlinear device or to translate one frequency to another using nonlinear mixing. Essentially there are five sections in a Superheterodyne receiver.

1. RF section
2. Mixer / converter section
3. IF section
4. Audio detector section
5. Audio amplifier section.

RF Section :-

The RF section generally consists of a preselector and an amplifier stage. They can be separate circuits or a single combined circuit. The preselector is a broad tuned bandpass filter with an adjustable centre frequency that is tuned to the desired carrier frequency. The primary purpose of the preselector is to provide enough initial bandlimiting to prevent a specific unwanted radio frequency called the image frequency. from entering the receiver. The preselector also reduces the noise bandwidth of the receiver and provides the initial step towards reducing the overall receiver bandwidth to the minimum bandwidth required to pass the information signal.

RF amplifier determines the sensitivity of a the receiver.

Advantages:-

1. Greater gain & better sensitivity
2. Improved image frequency rejection
3. Better Signal to Noise ratio
4. Better selectivity.

Mixer/converter section:-

The mixer / converter section includes a radio frequency oscillator stage and a mixer/converter stage. The mixer is a non-linear device and its purpose is to convert radio frequencies to intermediate frequencies. Heterodyning takes place in the mixer stage, and radio frequencies are down converted to intermediate frequencies. Althogh the carrier and sideband frequencies are translated from RF to IF., the shape of the envelope remains the same and ∴ the original information contained in the envelope remains unchanged. Bandwidth remain unchanged due to heterodyning. The intermediate frequency of AM is 455 kHz.

IF section : (APR/MAY 2017)

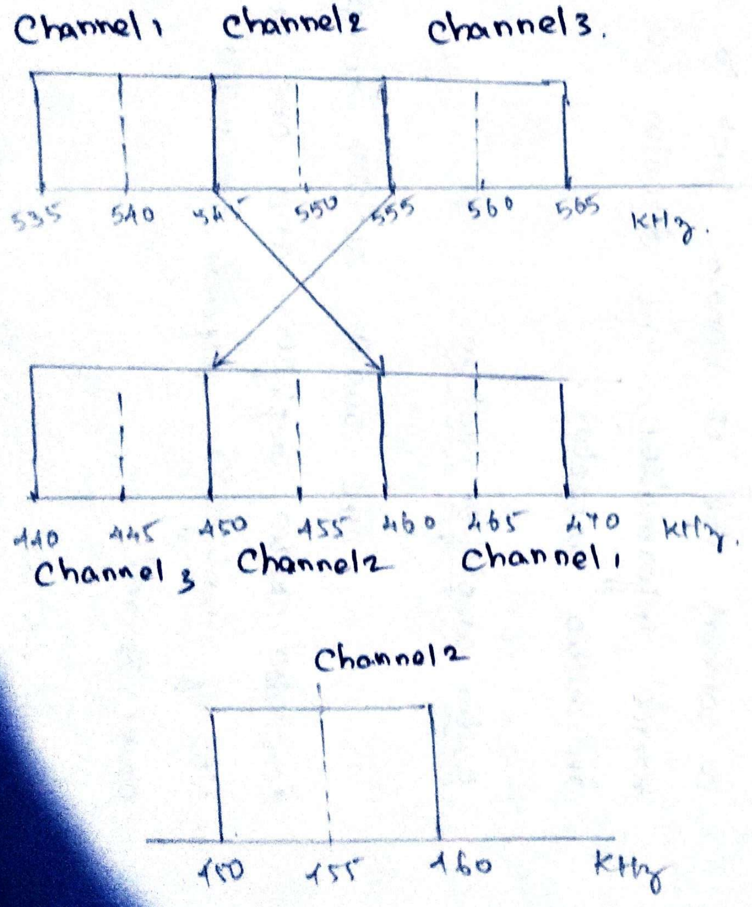
The IF section consists of a ~~seris~~ band pass filters, series of of IF amplifiers and ~~sensitivity~~ selectivity. Most of the receiver gain and ~~sensitivity~~ selectivity is achieved in the IF section. The IF centre frequency and bandwidth are constant for all stations and are chosen so that their frequency is less than any of the RF signals to be received. The IF is always lower in frequency than the RF because it is easier and less expensive to construct high gain stable amplifiers for the low frequency signals.

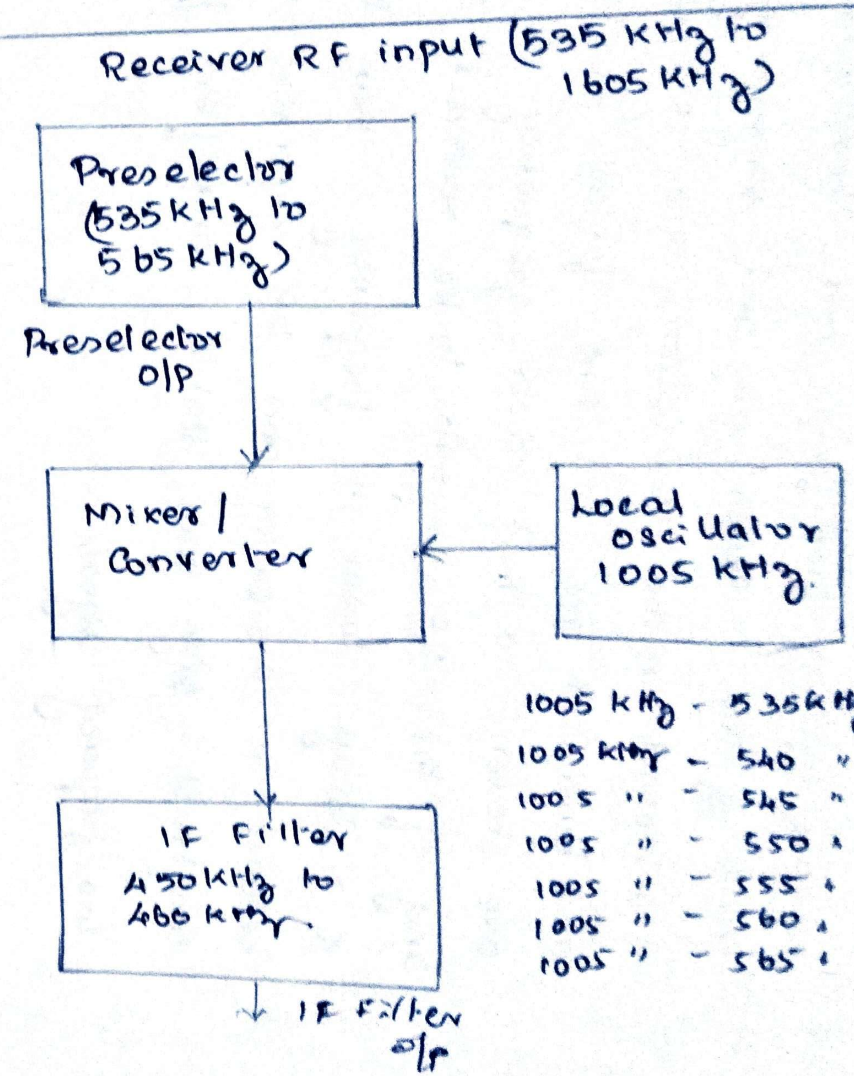
Detector section :

The purpose of the detector section is to convert the IF signals back to the original source information. The detector is generally called an audio detector.

Audio amplifier section :

The audio section comprises several cascaded audio amplifiers and one or more speakers. No: of amplifiers depends on audio signal power desired.





1005 KHz - 535 KHz : 470 KHz
1005 KHz - 540 " = 465 "
1005 " - 545 " = 460 "
} ch 1

1005 " - 550 " : 455 "
1005 " - 555 " : 450 "
} 2

1005 " - 560 " : 445 "
1005 " - 565 " : 440 "
} 3.

in the Hilbert

Receiver operation :

During the demodulation process in a superheterodyne receiver, the received signals undergo two or more frequency translations. First the RF is converted to IF, then the IF is converted to the source information.

RF for the Commercial AM broadcast band are frequencies between 535 kHz to 1605 kHz., and IF signals are frequencies between 450 kHz to 460 kHz. Intermediate frequencies refer to frequencies that are used within a transmitter or receiver that fall somewhere between the radio frequencies and the original source information frequencies.

Frequency Conversion :

Frequency conversion in the mixer/converter stage is identical to frequency conversion in the modulator stage of a transmitter except that in the receiver, the frequencies are down converted rather than up converted. In the mixer/converter, RF signals are combined with the local oscillator frequency in a non-linear device. The output of the mixer contains an infinite number of harmonics and cross product frequencies, which include the sum

The result of the Hilbert Transform

and difference frequencies between the desired RF carrier and local oscillator frequencies. The IF filters are tuned to the difference frequencies. The local oscillator is designed such that its frequency of osciulation is always above or below the desired RF carrier by an amount equal to IF centre frequency.

∴ Difference between the RF and local oscillator frequency is always equal to IF.

The adjustment for the centre frequency of the preselector and the adjustment for the local oscillator frequency are gang tuned. Gang tuning means that two adjustments are mechanically tied together. So that a single adjustment will change the centre frequency of preselector and at the same time, change the local oscillator frequency. When the local oscillator frequency is tuned above the RF it is called high side injection. When the local oscillator frequency is tuned below the RF it is called low side injection.

For high side injection $f_{lo} = f_{RF} + f_{IF}$

For low side injection $f_{lo} = f_{RF} - f_{IF}$.

The result of the Hilbert transform is

f_{lo} = Local oscillator frequency (Hz)

f_{RF} = radio frequency (Hz)

f_{IF} = Intermediate frequency (Hz).

The input to the receiver could contain any of the AM broadcast band channels, which occupy the bandwidth between 535 kHz to 1605 kHz.

In the example, the preselector is tuned to channel 2 which operates at 550 kHz, carrier frequency and contains side bands extending from 545 kHz to 555 kHz. The preselector is broadly tuned to a 30 kHz passband allowing channels 1, 2, and 3 to pass through it in to the mixer /converter stage., where they are mixed with a 1005 kHz local oscillator frequency. The mixer o/p contains the same three channels except because high side injection is used., the heterodyning process causes the side bands to be inverted. In addition, channels 1 and 3 switch places in the frequency domain with respect to channel 2.

The heterodyning process converts channel 1 from 535 kHz to 545 kHz band to 460 kHz to 470 kHz band.. channel 2 from 545 kHz to 555 kHz band. to 450 kHz to 460 kHz and channel 3 from. 555 kHz to 565 kHz band.

to 440 KHz to 450KHz band. Channel 2 is the only channel that falls within the bandwidth of IF Filters.

∴ Channel 2 is the only Channel that continues through the receiver to the IF amplifiers and eventually AM demodulator circuit.

Hilbert transform & its properties:- (APR/MAY 2018)

Hilbert transform is unlike many other transforms because it does not involve a change of domain. In contrast Fourier, Laplace and Z-transforms start from time domain representation of a signal and introduce the transform as an equivalent frequency domain representation of signals. The resulting two signals are equivalent representations of the same signal in terms of two different arguments. time and frequency.

The result of the Hilbert transform is not equivalent to the original signal, rather it is a completely different signal.

The Hilbert transform does not involve a domain change (ie) the Hilbert transform of a signal $x(t)$ is another signal denoted by $\hat{x}(t)$ in the same domain.

The Hilbert transform of a signal $x(t)$ is a signal $\hat{x}(t)$. whose frequency components lag the frequency components of $x(t)$ by 90°. In other words, $\hat{x}(t)$ has exactly the same frequency components present in $x(t)$ with the same amplitude except there is a 90° phase delay.

consider $x(t) = A \cos(2\pi f_0 t + \theta)$

The Hilbert transform of the above signal is,

$$\hat{x}(t) = A \cos(2\pi f_0 t + \theta - 90^\circ)$$

$$= A \sin(2\pi f_0 t + \theta).$$

A delay of $\frac{\pi}{2}$ at all frequencies means $e^{j2\pi f_0 t}$ will become $e^{j(2\pi f_0 t - \frac{\pi}{2})}$ $= -je^{j2\pi f_0 t}$ and $e^{-j2\pi f_0 t}$ will become

$$e^{-j(2\pi f_0 t - \pi/2)} = j\, e^{-j2\pi f_0 t}$$

At positive frequencies, the spectrum of the signal is multiplied by $-j$; and at negative frequencies, it is multiplied by $+j$. This is equivalent to saying that the spectrum of the signal is multiplied by, $-j\, sgn(f)$.

Assume that $x(t)$ is real and has no DC component (ie) $X(f)\,|_{f=0} = 0$

$$\therefore F[\hat{x}(t)] = -j\, sgn(f)\, X(f).$$

$$F[-j\, sgn(f)] = \frac{1}{\pi t}$$

Hence

$$\hat{x}(t) = \frac{1}{\pi t} * x(t) = \frac{1}{\pi t} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}\, d\tau.$$

Thus the operation of the Hilbert transform is equivalent to a Convolution (ie) filtering.

Performing the Hilbert transform on a signal is equivalent to a 90° phase shift in all its frequency components ∴ The only change that the Hilbert transform performs on a signal is changing its phase. Most important the amplitude of the frequency components of the signal and ∴ the energy and power of the signal donot change by performing the Hilbert transform operation.

Properties:

Evenness & oddness :-

The Hilbert transform of an even signal is odd and the Hilbert transform of odd signal is even.

If $x(t)$ is even, then $x(f)$ is a real and even function. ∴ $-j\,sgn(f)\,x(f)$ is an imaginary and odd function. Hence its inverse Fourier transform $\hat{x}(t)$ will be odd.

If $x(t)$ is odd, then $x(f)$ is imaginary and odd. Thus $-j\,sgn(f)\,x(f)$ is real and even. ∴ $\hat{x}(t)$ is even.

Sign Reversal :

Applying the Hilbert transform operation to a signal twice causes a sign reversal of the signal.

(ii) $\hat{\hat{x}}(t) = - x(t)$

$$\Rightarrow \quad F[\hat{\hat{x}}(t)] = [-j \operatorname{sgn}(f)]^2 X(f)$$

$$\Rightarrow \quad F[\hat{\hat{x}}(t)] = - X(f).$$

where $X(f)$ does not contain any impulses at the origin.

Energy : The energy content of a signal is equal to the energy content of its Hilbert transform.

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt \Leftrightarrow \int_{-\infty}^{\infty} |X(f)|^2 df.$$

$$\text{llly} \quad \hat{E}_x = \int_{-\infty}^{\infty} |\hat{x}(t)|^2 dt \Leftrightarrow \int_{-\infty}^{\infty} |-j \operatorname{sgn}(f) X(f)|^2 df.$$

$$|-j \operatorname{sgn}(f)|^2 = 1$$

$$\Rightarrow \hat{E}_x = \int_{-\infty}^{\infty} |X(f)|^2 df = E_x.$$

Orthogonality :

The signal $x(t)$ and its Hilbert transform are orthogonal

Using Parsevals theorem of the Fourier transform,

$$\int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = \int_{-\infty}^{\infty} X(f)\left[-j\,\mathrm{sgn}\,X(f)\right]^{*} df$$

$$= -j\int_{-\infty}^{0} |X(f)|^{2} df + j\int_{0}^{\infty} |X(f)|^{2} df.$$

$$= \underline{\underline{0}}\,..$$

Pre Envelope & Complex Envelope :

Pre envelope :

The Pre-Envelope of the signal $x(t)$ is defined as,

$$x_p(t) = x(t) + j\,\hat{x}(t).$$

$x(t)$ is the real part of pre-envelope and Hilbert transform $\hat{x}(t)$ is the imaginary part of pre envelope.

The Fourier transform of the pre envelope is given as,

$$X_p(f) = X(f) + j\left[-j\,sgn(f)\,X(f)\right]$$

$$= X(f) + Sgn(f)\,X(f).$$

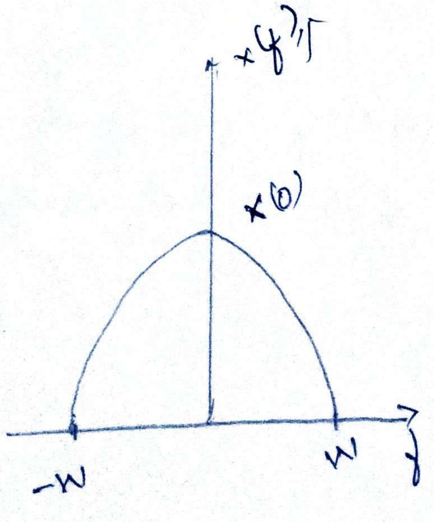
we know that,

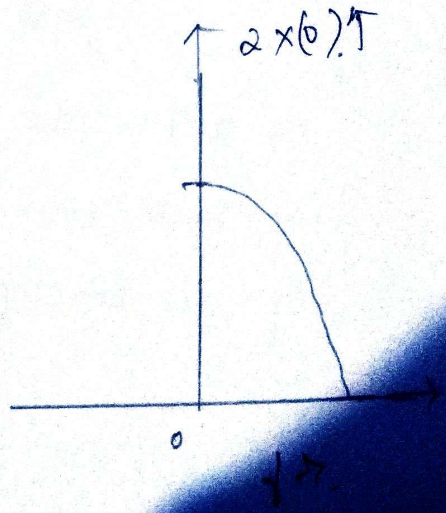
$$Sgn(f) = \begin{cases} 1 & for\ f>0 \\ 0 & for\ f=0 \\ -1 & for\ f<0. \end{cases}$$

above equation becomes,

$$X_p(f) = \begin{cases} 2X(f) & for\ f>0 \\ X(0) & for\ f=0 \\ 0 & for\ f<0. \end{cases}$$

Thus the Pre-envelope of the signal has no frequency content for negative freq.





Complex envelope :-

The complex envelope of the bandpass signal $x(t)$ is given as,

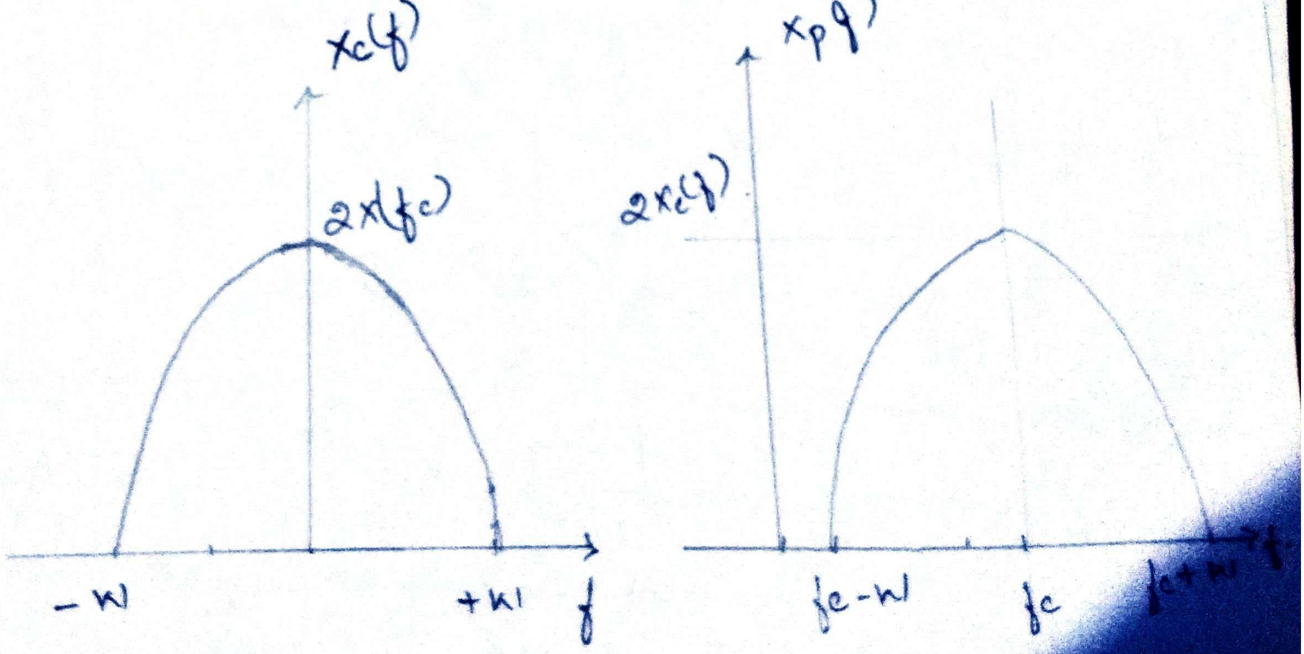
$x_c(t) = x_p(t)\, e^{-j2\pi f_c t}$

Here $x_c(t)$ is the complex envelope and $x_p(t)$ is the pre envelope. f_c is the center frequency of the band pass signal.

$\therefore\ x_p(t) = x_c(t) - e^{j2\pi f_c t}$

Fourier transform of above equation becomes

$x_p(f) = x_c(f - f_c)$.

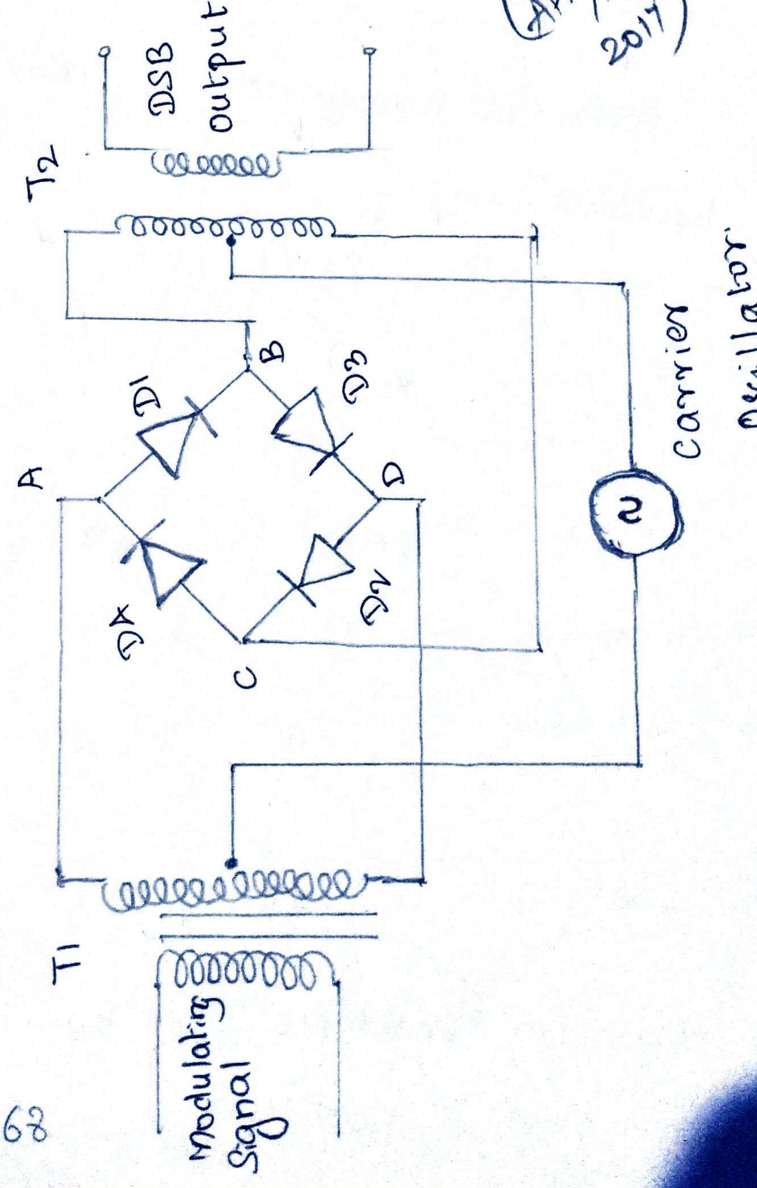


Complex envelope of the bandpass signal & low pass spectrum.

Suppression of the carrier :

The balanced modulator is used to suppress the carrier from the AM signal. The inputs to the balanced modulator are carrier and modulating signal. The output of a balanced modulator is upper and lower sidebands with suppressed carrier (or) DSBSC signal.

Balanced Modulator or Ring modulator using diodes (APR/MAY 2017)



The sum of any two frequency components in the range $f_c - f_v \le f \le f_c + f_v$ is equal to unity

$$H(f - f_c) + H(f + f_c) = 1.$$

Phase response is linear.

Transmission Bandwidth.

$$B_T = f_v + W.$$

Advantages :

1. Low frequencies, near f_c are transmitted without any attenuation

2. Bandwidth is reduced compared to DSB.

Applications :

VSB is mainly used for TV transmission, since low frequencies near f_c represents significant picture details. They are unaffected due to VSB.

Comparison of various AM systems.

S.No.	Parameter	AM with Carrier	DSB -SC	SSB -SC	VSB.
1.	Method	Carrier & Both sidebands	Only Side bands	only one sideband	one sideband & part of other sideband
2.	Bandwidth	$2f_m$	$2f_m$	f_m	$f_m < BW < 2f_m$
3.	Generation	Easy	Easy	Complex	Complex.
4.	Transmission Efficiency	33.3%	100%	100%.	$33.3\% < \eta < 100$
5.	Selective fading	Heavy distortion	More distortion compared to SSB	Least distortion	Received signal is distorted

Performance Parameters of AM Receivers

1. Selectivity
2. Sensitivity
3. Fidelity
4. Image frequency Rejection

Unit-II. Angle Modulation

Phase and frequency Modulation - Narrowband and wide band FM - Modulation Index, Spectra, ~~FM~~ ~~&~~. Modulation - Power relations and bandwidth - FM Direct and Indirect methods, FM demodulation, FM to AM conversion, FM Discriminator, PLL as FM demodulator.

Introduction.

Angle modulation is a method of analog modulation in which either the phase or frequency of the carrier wave is varied according to the message signal. In this method of modulation, the amplitude of the carrier wave is remained constant.

Advantages:

1) Improved Noise Immunity and Interference.
2) Improved system fidelity and efficient power.

Properties of Angle-Modulated wave:-

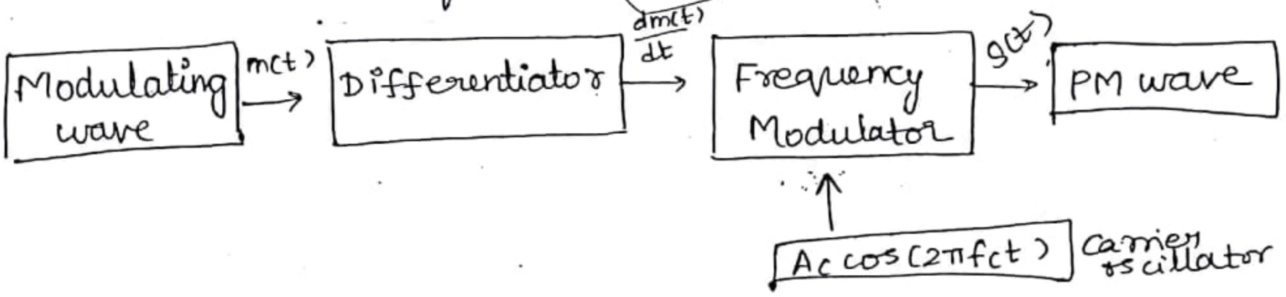
1. Constancy of transmitted power
2. Non linearity of modulation process.
3. Irregularity of zero crossing.
4. Visualization difficulty of message waveform.
5. Trade off of increased transmission bandwidth for improved noise performance.

Phase modulation:-

It is defined as the process by which phase of a carrier is varied in accordance with instantaneous value of modulating voltage or message signal, but frequency and amplitude remains same

Generation of Phase Modulation :-

It can be generated by differentiating the modulating signal m(t) and the differentiated output is used as the input of FM modulator.



Mathematical Expression [Representation] of PM :-

The phase modulated signal has the angle, $\theta(t)$ defined by,

$$\boxed{\theta(t) = 2\pi f_c t + \phi(t)} \rightarrow ①$$

$\phi(t)$ → instantaneous Phase deviation

$\theta(t)$ → Angle.

The $\phi(t)$ is directly proportional to the message s/g m(t)

$$\phi(t) \propto m(t)$$

$$\boxed{\phi(t) = K_p m(t)} \rightarrow ②$$

K_p → phase sensitivity of modulator in radians per volt.

Substitute ② in ①,

$$\theta(t) = 2\pi f_c t + k_p m(t) \rightarrow ③$$

The phase modulated signal is defined as,

$$s(t) = A_c \cos(2\pi f_c t + k_p m(t))$$

where $m(t) = V_m \cos\omega_m t$.

$$\therefore s(t) = A_c \cos(2\pi f_c t + k_p V_m \cos\omega_m t)$$

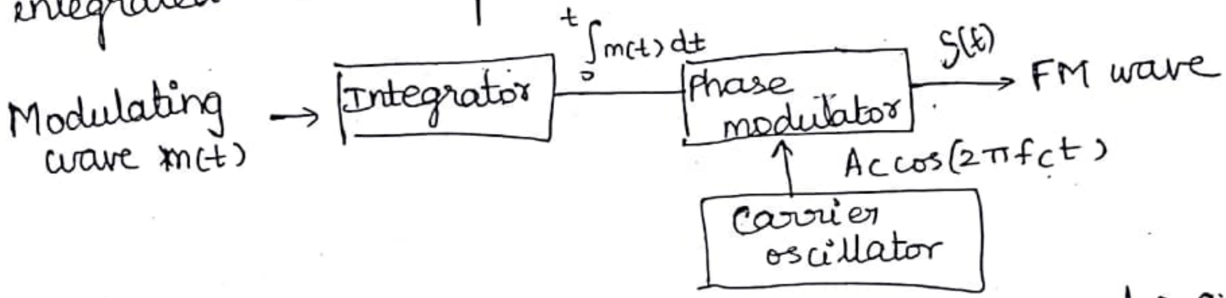
$$\boxed{\phi_m = K_p V_m}$$ ⇒ Modulation Index (or) Peak phase deviation.

Frequency Modulation :-

It is the process in which the instantaneo frequency f(t) is varied in linear proportion with the instantaneous magnitude of message signal m(t).

Generation :-

FM wave can be generated by applying the integrated version of m(t) to a phase modulator.



The instantaneous frequency f(t) of FM signal is given

$$\boxed{f(t) = f_c + k_f m(t)} \rightarrow ①$$

→ Frequency sensitivity

→ Frequency of unmodulated Carrier.

A complete oscillation occurs whenever θ(t) changes by 2π radians ⇒ $\boxed{f(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}} \rightarrow ②$

compare ① & ②,

$$\frac{1}{2\pi}\left(\frac{d\theta(t)}{dt}\right) = f_c + k_f m(t)$$

$$\frac{d\theta(t)}{dt} = 2\pi f_c + 2\pi k_f m(t).$$

Applying integration on both sides.

$$\int \frac{d\theta(t)}{dt} = \int_0^t 2\pi f_c\,dt + \int_0^t 2\pi k_f m(t)\,dt.$$

$$\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt.$$

$$= 2\pi f_c t + 2\pi k_f V_m \int_0^t \cos\omega_m t\,dt.$$

$$\theta(t) = 2\pi f_c t + \frac{k_f V_m}{f_m}\sin 2\pi f_m t.$$

$\boxed{\Delta f = k_f V_m}$

$\hookrightarrow$ frequency Deviation.

where $\boxed{\beta = \Delta f / f_m}$ ⇒ Modulation Index.

The equation for FM wave is given by, $s(t) = A_c \cos \theta(t)$

$$S(t) = A_c \cos [\omega_c t + \beta \sin \omega_m t].$$

Modulation Index of FM :-

$$\beta = \frac{\Delta f \rightarrow \text{frequency deviation}}{f_m \rightarrow \text{modulating frequency}}.$$

- It decides the Bandwidth of FM, and the numbe of sidebands having significant Amplitudes.
- The value of modulation Index can be greater than 1 (Note: For AM, m_a lies b/n 0 to 1).

Deviation Ratio:-

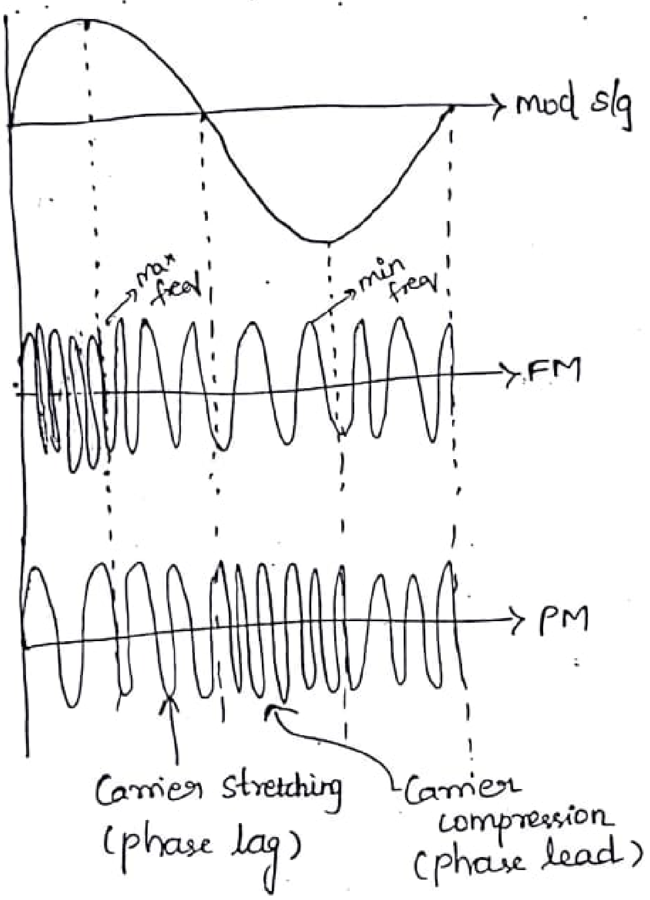
The modulation Index corresponding to the maximum deviation (limited to 75KHz) and the maximum modulating Frequency (limited to 15KHZ). It is called as the Deviation Ratio.

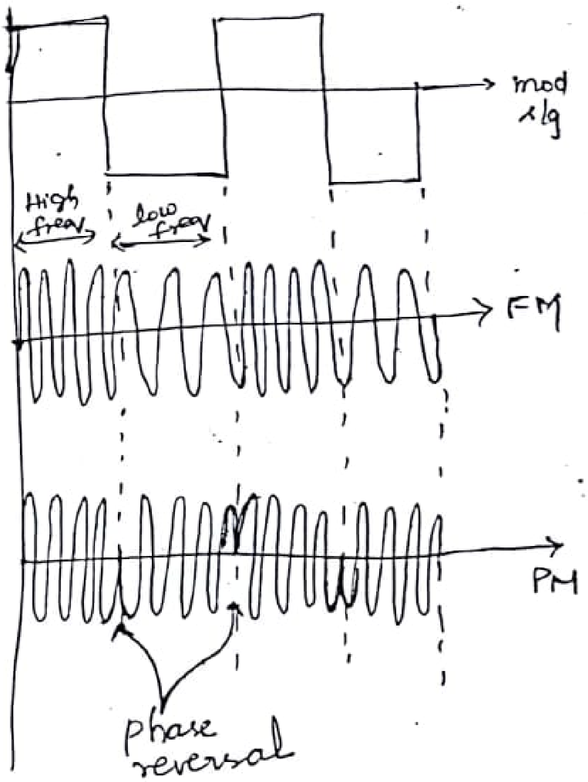
$$D.R = \frac{\text{Maximum Deviation}}{\text{Maximum Modulating frequency}}. = \frac{75k}{15k} = 5$$

Percentage Modulation :-

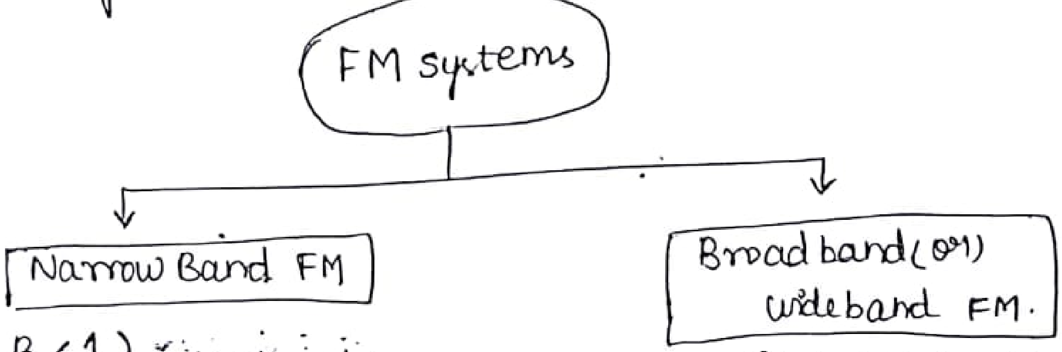
$$\%\text{ Modulation} = \frac{\text{Actual freq. deviation}}{\text{Maximum Allowed deviation}}$$

Waveform Representation





Depending on modulation Index, FM can be classified as



Narrow Band FM:

($\beta < 1$)

(Δf = 5KHZ)

(f_m = 30Hz to 3KHZ)

(Also called low Index FM)

Broad band (or) wideband FM:

($\beta > 1$) ≃ (5 to 2500)

(Δf = 75KHz)

(f_m = 30Hz to 15kHz)

(Also called high index FM).

Narrow Band FM :-

If the modulation Index is less than one $(\beta << 1)$ then it is called Narrow Band FM.

$$s(t) = A_c \cos\underbrace{[2\pi f_c t + \beta \sin(2\pi f_m t)]}$$

$$= A_c \quad \cos(A+B)$$

$$s(t) = A_c \cos(2\pi f_c t)\cos(\beta.\sin 2\pi f_m t) - A_c \sin(2\pi f_c t).\ \sin(\beta \sin(2\pi f_m t));$$

For Narrow band FM signal $\beta << 1$;

$$\cos(\beta \sin(2\pi f_m t)] \simeq 1$$
$$\sin(\beta \sin(2\pi f_m t)] \simeq \beta \sin(2\pi f_m t) \quad \{\because \sin\theta \simeq \theta\}.$$

$$\therefore\ s(t) = A_c \cos 2\pi f_c t - A_c \underbrace{\sin 2\pi f_c t\ (\beta \sin(2\pi f_m t))}_{\sin A \sin B}.$$

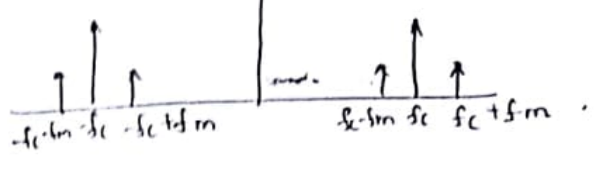
$$s(t) = A_c \cos 2\pi f_c t - \frac{A_c\beta}{2}\left[\cos 2\pi(f_c - f_m)t - \cos 2\pi(f_c + f_m)t\right]$$

$$s(t) = A_c \cos 2\pi f_c t + \frac{A_c\beta}{2}\cos(2\pi\underbrace{(f_c+f_m)}_{USB}t) - \frac{A_c\beta}{2}\cos 2\pi\underbrace{(f_c-f_m)}_{LSB}t$$

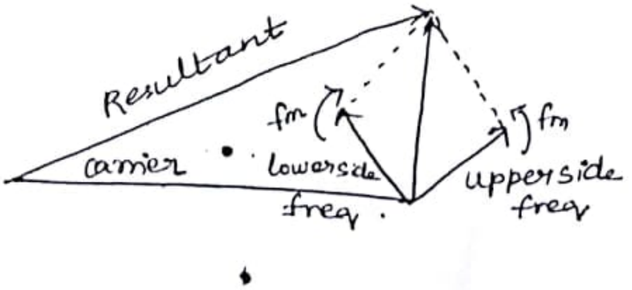
→ 180° phase shift.

Narrow band FM is mainly used in FM mobile communicat such as police wireless, Ambulances, taxicabs etc.,

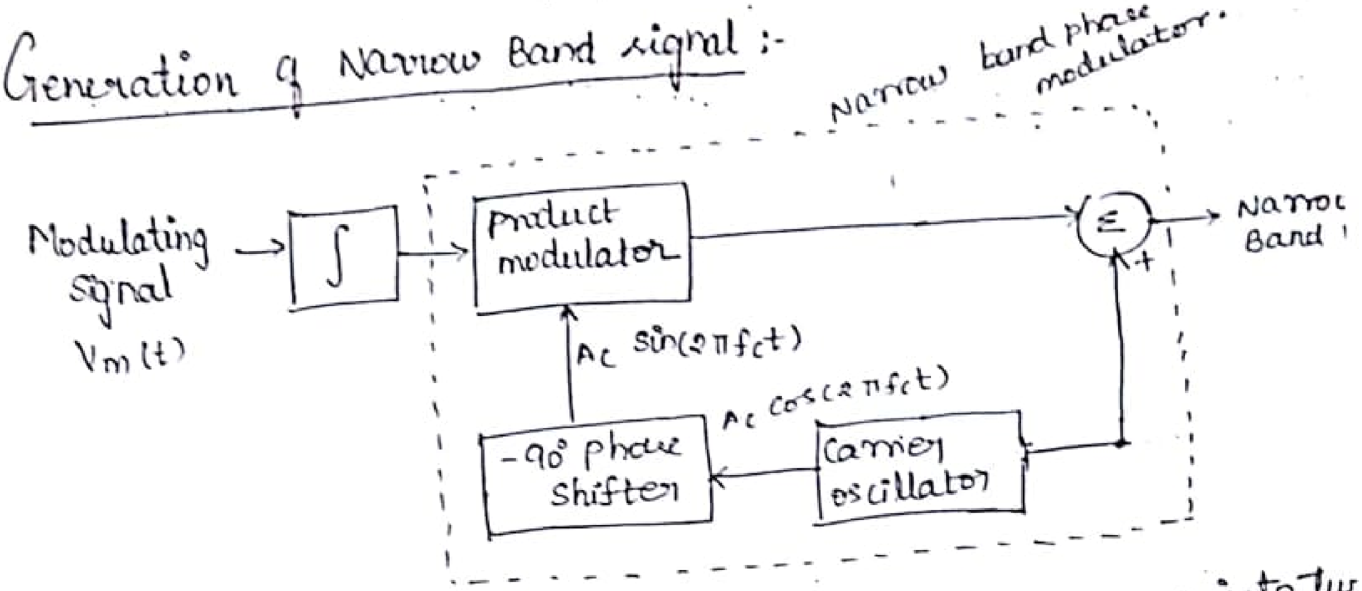
Magnitude Spectrum



Phasor Diagram



Generation of Narrow Band signal :-



This modulator involves the splitting of carrier wave into tw paths. one path is direct and other path contains -90° phase shifting network and a product modulator, the combination whic generates a DSB-SC modulated signal.

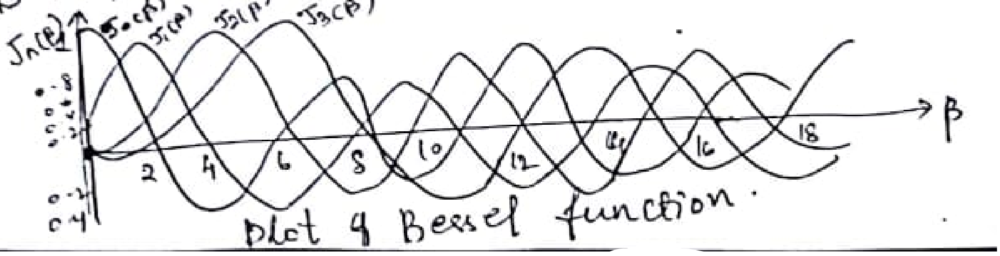
The difference between these two signals produces a Narrow band FM signal, but with some distortion.

Ideally, an FM signal has a constant envelope and for the case of a sinusoidal modulating signal of frequency fi the angle $\phi_i(t)$ is also sinusoidal with the same frequency.

Wideband FM :-

FM wave ideally contains the carrier and an infinite number of sidebands located symmetrically around the carrier. such an FM wave has infinite Bandwidth and hence called wideband FM. It is mainl used in entertainment Broadcasting Application such FM radio, TV etc.)

-If $\beta >> 1$ then it is called as wideband FM.



Plot of Bessel function.

It can be obtained by multiplying the narrow band FM signal by using suitable Frequency multiplier.

The resultant FM s/g is given by,

$$s(t) = A_c[\overset{sin}{\cos}(2\pi f_c t) + \beta\ \overset{cos}{\sin}(2\pi f_m t)] \rightarrow ①$$

The phase angle of FM,

$$\theta(t) = [2\pi f_c t + \beta\ \overset{cos}{\sin}(2\pi f_m t)] \rightarrow ②$$

The FM wave can be expressed in terms of complex envelope as,

$$s(t) = Re\{A_c\ e^{j\theta(t)}\}$$

$$= Re\{A_c\ e^{j(2\pi f_c t + \beta \overset{cos}{\sin} 2\pi f_m t)}\}$$

$$s(t) = Re\{A_c\ e^{j\omega_c t}\ e^{j\beta \overset{cos}{\sin}\omega_m t}\} \rightarrow ③$$

Mathematical expression of Fourier series,

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{jnx} \rightarrow ④$$

↳ Fourier coefficient

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) e^{-jnx}\,dx \rightarrow ⑤$$

The Second exponential term in eq ③ can be expanded in Fourier series,

$$e^{j\beta \overset{cos}{\sin}\omega_m t} = \sum_{n=-\infty}^{\infty} c_n\ e^{jn\omega_m t}$$

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\beta \overset{cos}{\sin}\omega_m t}\ e^{-jn\omega_m t}\,dt$$

put $\boxed{\omega_m t = x}$ $dt = dx$

$$\Rightarrow c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\beta \overset{cos}{\sin}x}\ e^{-jnx}\,dx \rightarrow ⑥$$

$$c_n = J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta \overset{cos}{\sin}x - nx)}\,dx \rightarrow ⑦$$

where $J_n(\beta)$ is the Bessel function of first kind of order n.

$$e^{j\beta \sin \omega_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn\omega_m t} \rightarrow ⑧$$

substitute ⑧ in ③ we get,

$$s(t) = Re\left\{ A_c e^{j\omega_c t} \cdot \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn\omega_m t} \right\}$$

$$= Re\left\{ A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j(\omega_c + n\omega_m)t} \right\}$$

$$= Re\left\{ A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi(f_c + nf_m)t} \right\}.$$

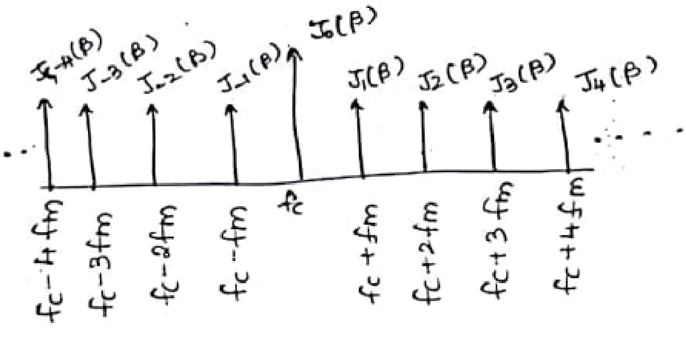
$$\boxed{s(t) = A_c \cdot \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + nf_m)t)}$$

Table. whe: (f_c+nf_m) $n \geq F$

Modulation index	no of sideb
β	$2n_{ina}$
0.1	2
0.3	4
0.5	4
1.0	6
2.0	8
5.0	16
10.0	28
20.0	50
30.0	70

The above equation representing Fourier series of the single tone of FM signal. It has infinite number of sidebands at frequencies $(f_c + nf_m)$.

Magnitude Spectrum

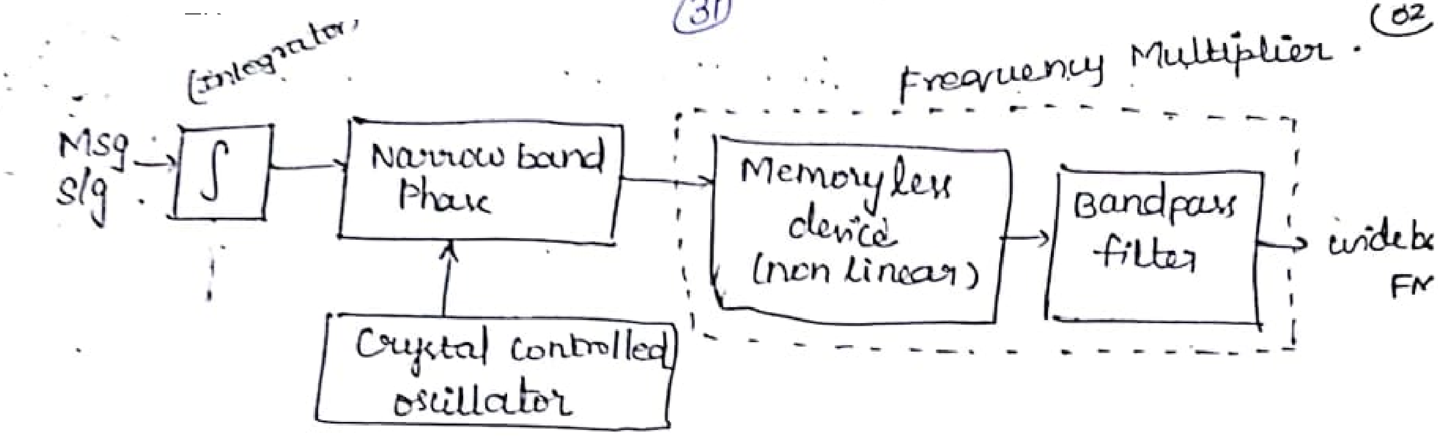


properties of Bessel function

1. $J_n(\beta) = (-1)^n J_{-n}(\beta)$ for all n
2. $J_{n+1}(\beta) + J_{n-1}(\beta) = (2n/\beta) J_n(\beta)$
3. $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$
4. $J_0(\beta) \approx 1$, $J_1(\beta) \approx (\beta/2)$, $J_2(\beta) \approx 0$ for $n > 2$.

Generation of wideband signal :-

The message s/g is given to integrator. crystal oscillator generates carrier s/g and gives to phase modulato The Frequency multiplier converts Narrow band FM to wideba FM by a Nonlinear device may be diode or transistor.

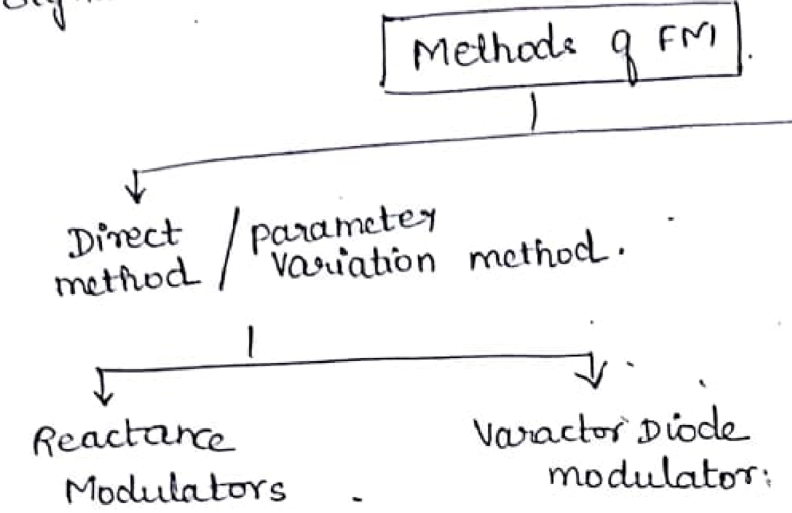


Comparison of Narrowband and wideband FM

Sno	Parameter/ Characteristics	wideband FM	Narrowband FM.
1.	Modulation Index	Greater than 1	less than (or) slightly greater than 1.
2.	Maximum Deviation	75KHz	5KHz.
3.	Range of Modulating frequency	30Hz to 15KHz.	30Hz to 3KHz.
4.	Bandwidth	Large, about 15 times higher than B.W of narrow band. $B.W = 2(\Delta f + f_m)$	small. Approximately same as that of AM $BW = 2f_m$.
5	Maximum modulation Index	5 to 2500	slightly greater than 1
6.	pre-emphasis & De-emphasis	Needed	Needed.
7.	Noise	Noise is more suppres	Less suppressing of noise.
8.	Applications	Entertainment broadcasting.	FM mobile communicati like police. wireless, ambulances.
9	Side bands	∞ S.B & carrier.	two Side bands & carrier
10	Expression	$V_{FM}(t) = V_c \sum_{n=-\infty}^{\infty} J_n(m_f) \cos(\omega_c t + n\omega_m t)$	$V_{FM}(t) = V_c \sin\omega_c t + m_f V_c \cos\omega_c t \sin\omega_m t$.

Frequency Modulation (or) Frequency Generation

The FM modulator circuits used for generating FM signals.



Direct Method :-

The baseband or modulating signal directly modulates the carrier. The carrier signal is generated by an oscillator circuit. This circuit uses a parallel tuned L-C circuit. Thus frequency q oscillation q carrier is governed by the expression $\boxed{\omega_c = \frac{1}{\sqrt{LC}}}$. In oscillator circuit frequency is controlled by a modulating voltage is called Voltage controlled oscillator (VCO)

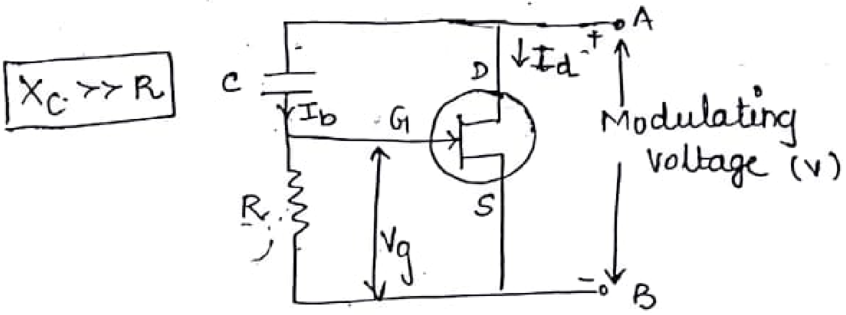
Reactance Tube Modulator :-

A Reactance modulator is an Amplifier that made to appear Inductive or capacitive by phase shift: used to produce wide-deviation direct FM.

A transistor or FET is operated as a variable reactance (L or C). This device is connected across the tuned circuit q an oscillator. As the instantaneous value q modulating voltage changes, the reactance offered by the Transistor or FET will change proportionally. This will change t

Frequency of oscillator to produce FM wave.

Basic FET Reactance Modulator :-



Assumption :-

(i) Bias Network current Ib is negligible as compared to the drain current of FET.

(ii) Drain to gate impedance (Xc) must be greater than gate to source Impendance (R). ie., $\boxed{X_C >> R}$

The above circuit represents basic FET reactance modulator. It behaves reactance across terminals A - B. It may be connected across the tuned circuit of the oscillator to get FM output.

The value of this reactance is proportional to the transconductance g_m of the FET, which can be made depend on gate bias and its variation.

Expression :-

Gate voltage $V_g = I_b \cdot R$ (or)

(By Vtg divider rule) $V_g = \frac{R.V.}{(R - jX_C)} \longrightarrow ①$

Drain current $I_d = g_m \times V_g$.

$I_d = g_m \left(\frac{RV}{R - jX_C} \right) \longrightarrow ②$

Assuming that I_b is very small as compared to I_d

Impedance between terminals AB } $Z = \frac{V}{I_d}$.

$$Z = \frac{\cancel{V}}{\frac{g_m R \cancel{V}}{(R - jX_c)}}$$

$$Z = \frac{R - jX_c}{g_m \cdot R}.$$

$$Z = \frac{1}{g_m}\left(1 - \frac{jX_c}{R}\right) \rightarrow ③.$$

If $\boxed{X_c >> R}$ $\Rightarrow$ $Z = \frac{-jX_c}{g_m R} \rightarrow ④$

Eq ④ clearly represents a capacitive Reactance

$$Z = X_{eq} = \frac{X_c}{g_m \cdot R} = \frac{1}{2\pi f g_m RC}$$

$$Z = \frac{1}{2\pi f C_{eq}} \rightarrow ⑤.$$

where $C_{eq} = g_m RC. \rightarrow ⑥.$

This expression shows that FET is equivalent to a variable capacitance C_{eq}.

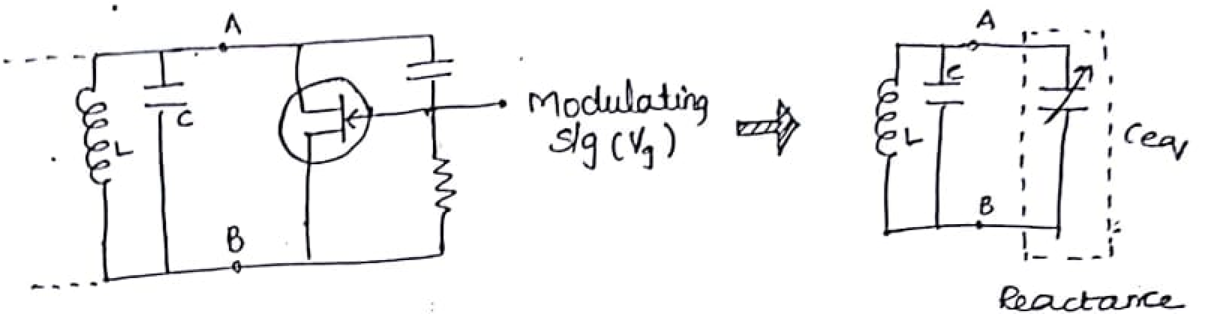
In practice, $\boxed{X_C = nR}$ at carrier frequency.

$$X_C = \frac{1}{\omega C} = nR.$$

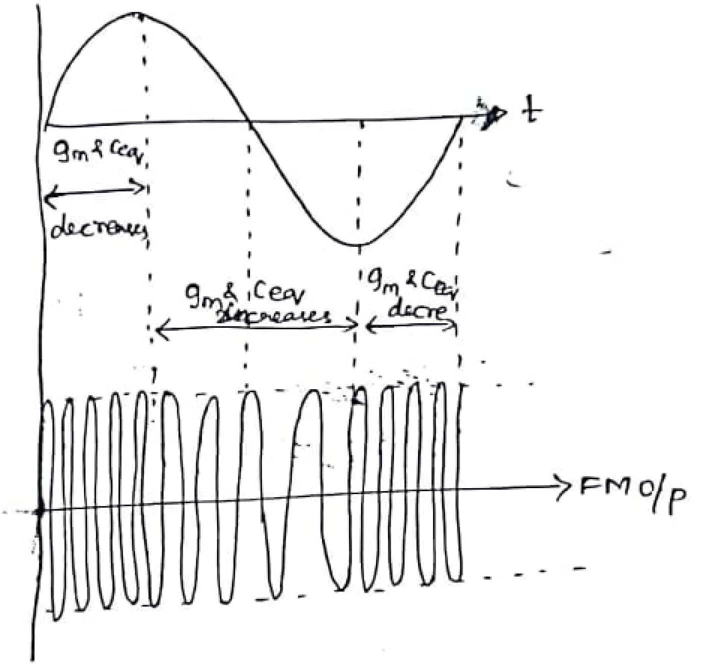
$$\therefore C = \frac{1}{\omega n R} = \frac{1}{2\pi f n R} \rightarrow ⑦$$

substitute ⑦ in ⑥ $\therefore$ $C_{eq} = \frac{g_m \cancel{R}}{2\pi f n \cancel{R}} = \frac{g_m}{2\pi f n}$.

The modulating voltage is applied ~~to~~ at the gate, and the terminal A-B are connected across LC resonant circuit



Reactance modulator act as variable



As V_g increases
↓
g_m decreases
↓
C_{eq} decreases
↓
Frequency Increases.

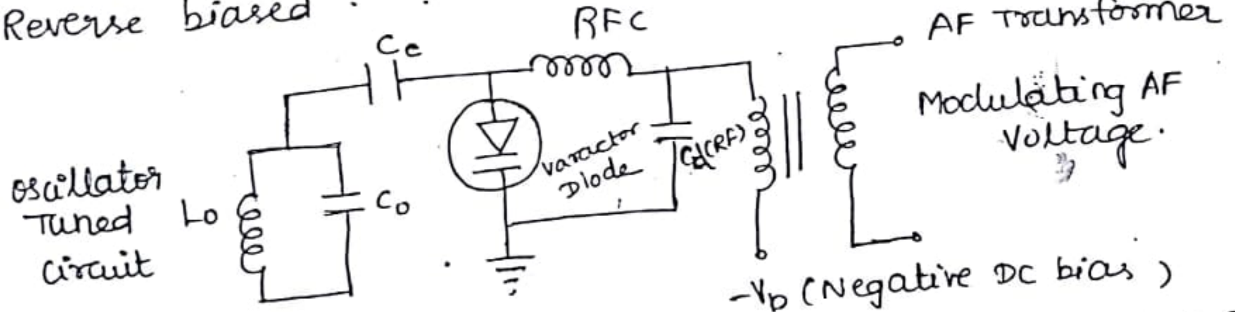
Types of Reactance Modulator:-

Sno	Name	ZGD	ZGS	condition	Reactance Formula.
1	RC capacitive	C	R	$X_C >> R$	$C_{eq} = g_m RC$
2	RC Inductive	R	C	$R >> X_C$	$L_{eq} = RC/g_m$
3	RL Inductive	L	R	$X_L >> R$	$L_{eq} = L/g_m R$
4	RL Capacitive	R	L	$R >> X_L$	$C_{eq} = g_m L/R$.

ZGD → Impedance of gate to Drain
ZGS → Impedance of gate to Source.

Varactor Diode Modulator :-

A Varactor Diode [Variable capacitor or Varicap] is a Semiconductor diode whose Junction capacitance varies linearly with the applied bias. the diode must be "Reverse biased".



The coupling capacitor isolates the varactor diode from the oscillator as far as D.C bias is concerned while providing an effective short circuit at operating frequencies.

The modulating AF voltage appears in series with the negative supply voltage. Hence the Voltage applied across the varactor diode varies in proportion with modulating voltage. This will vary the Junction Capacitance of the varactor diode. The varactor diode appears in parallel with oscillator tuned circuit.

Here the oscillator frequency will change with change in varactor diode capacitance and FM wave is produced. The RFC will connect the dc and modulating signal to the varactor diode but it offers a very high impedance at high oscillator frequency. ∴ The oscillator circuit is isolated from the dc bias and modulating signal.

The capacitance of the diode Cd is given by,

$$C_d = K(V_D)^{-1/2}$$

↳ Total instantaneous Vtg across Diode.

→ constant of proportionality.

RFC → Radio Frequency choke.

The expression for V_D is given by

$$V_D = V_b + \text{Modulating s/g}$$

$$= V_b + V_m \sin\omega_m t$$

↳ polarizing Vtg to maintain reverse bias.

In oscillator Tank circuit,

$$\text{Total capacitance} = C_o + C_d$$

$$\text{Frequency of oscillation} = \omega_i = \frac{1}{\sqrt{L_o(C_o + C_d)}}$$

$$\boxed{\omega_i = \frac{1}{\sqrt{L_o(C_o + KCV_D)^{-1/2}}}}$$

∴ oscillator frequency ω_i is dependent on the modulating signal and thus Frequency modulation is generated.

Application:-

(i) Automatic frequency control

(ii) Remote tuning

Drawbacks of Direct method of FM generation:-

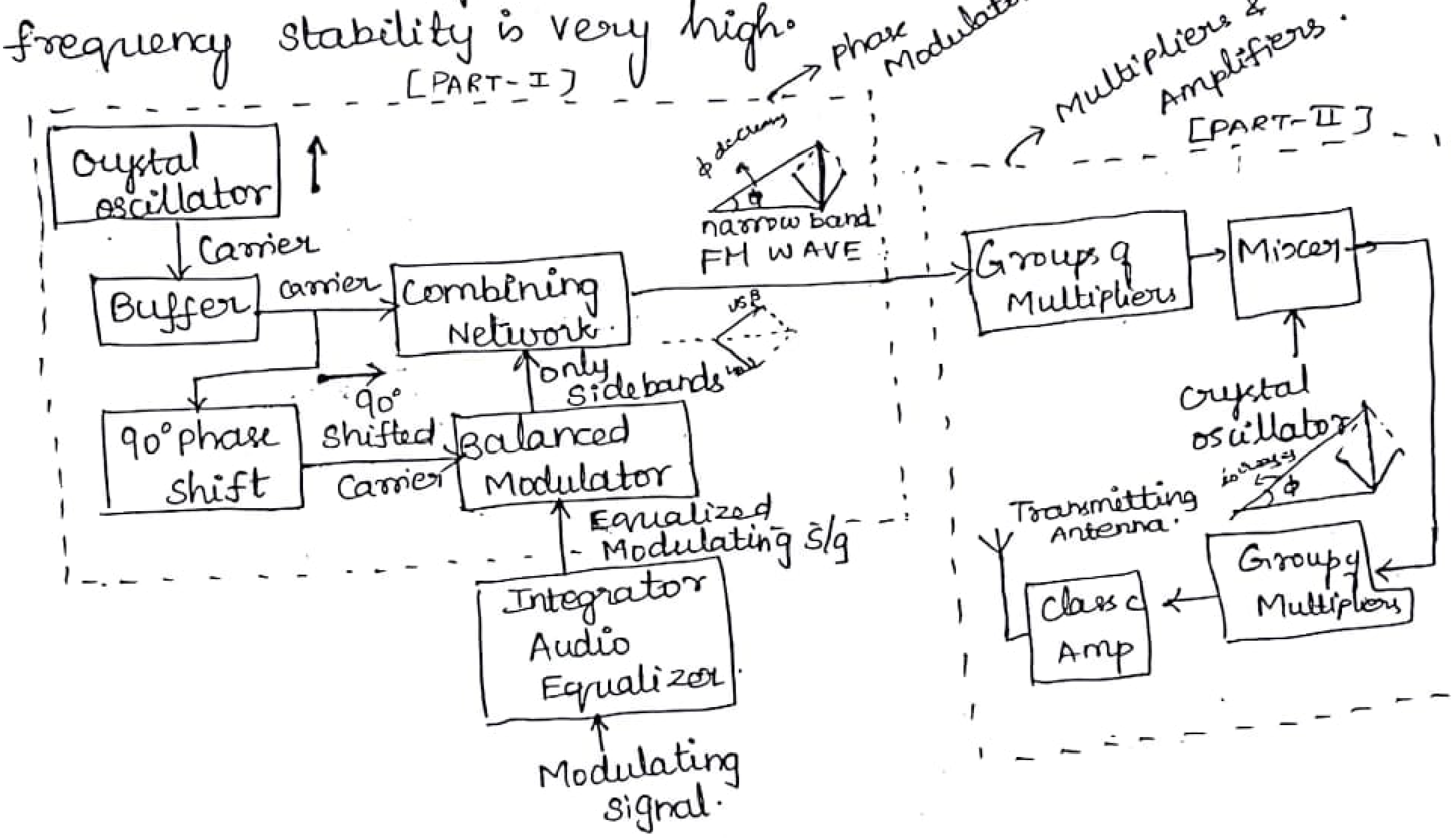
(i) Carrier generation cannot be high stability which is a necessary requirement.

(ii) Non-linearity of the varactor diode produces frequency variatic due to harmonics of modulating or baseband signal. ∴ FM signal is distorted.

—— × ——

Indirect Method [Armstrong Method] of FM Generation :-

The FM is obtained through phase modulatio n a phase modulator, carrier is shifted in phase in accordance with the modulating signal. This produces in direct FM. A crystal oscillator can be used hence the frequency stability is very high.



Phase Modulator :-

Generate a narrow band FM wave using a phase modulator. Modulating signal is integrated and the phase modulated with carrier signal.

Multiplier & Amplifiers :-

To obtain the required values of frequency deviation, carrier and modulation index. The multiplication process is performed in several stages in order to increase the carrier frequency as well as frequency deviation to the assigned value.

PART-I: Generate a Narrow band FM using Phase modulator.

Let the Narrow band FM wave produced at the output of phase modulator be represented by $s_1(t)$.

$$s_1(t) = V_{c1} \cos[2\pi f_1 t + \phi_1(t)] \longrightarrow ①$$

The phase angle $\phi_1(t) = 2\pi k_1 \int_0^t x(t)\,dt \longrightarrow ②$

$\hookrightarrow$ frequency sensitivity of the modulator.

Eq ① in the form of $\cos(A+B)$.

$$s_1(t) = V_{c1} \cos[2\pi f_1 t + \phi_1(t)]$$

$$= V_{c1}[\cos(2\pi f_1 t)\cos\phi_1(t) - \sin(2\pi f_1 t)\sin\phi_1(t)].$$

If $\phi_1(t)$ is small then, $\cos\phi_1(t) \simeq 1$; $\sin\phi_1(t) = \simeq \phi_1(t)$.

$$S_1(t) = V_{c1}\{\cos(2\pi f_1 t)(1) - \sin(2\pi f_1 t)\,\phi_1(t).$$

$$s_1(t) = V_{c1}\cos(2\pi f_1 t) - V_{c1}\,\phi_1(t)\sin(2\pi f_1 t)$$

$$s_1(t) = V_{c1}\cos 2\pi f_1 t - V_{c1}\sin 2\pi f_1 t\left[2\pi k_1 \int_0^t x(t)\,dt\right] \rightarrow ③.$$

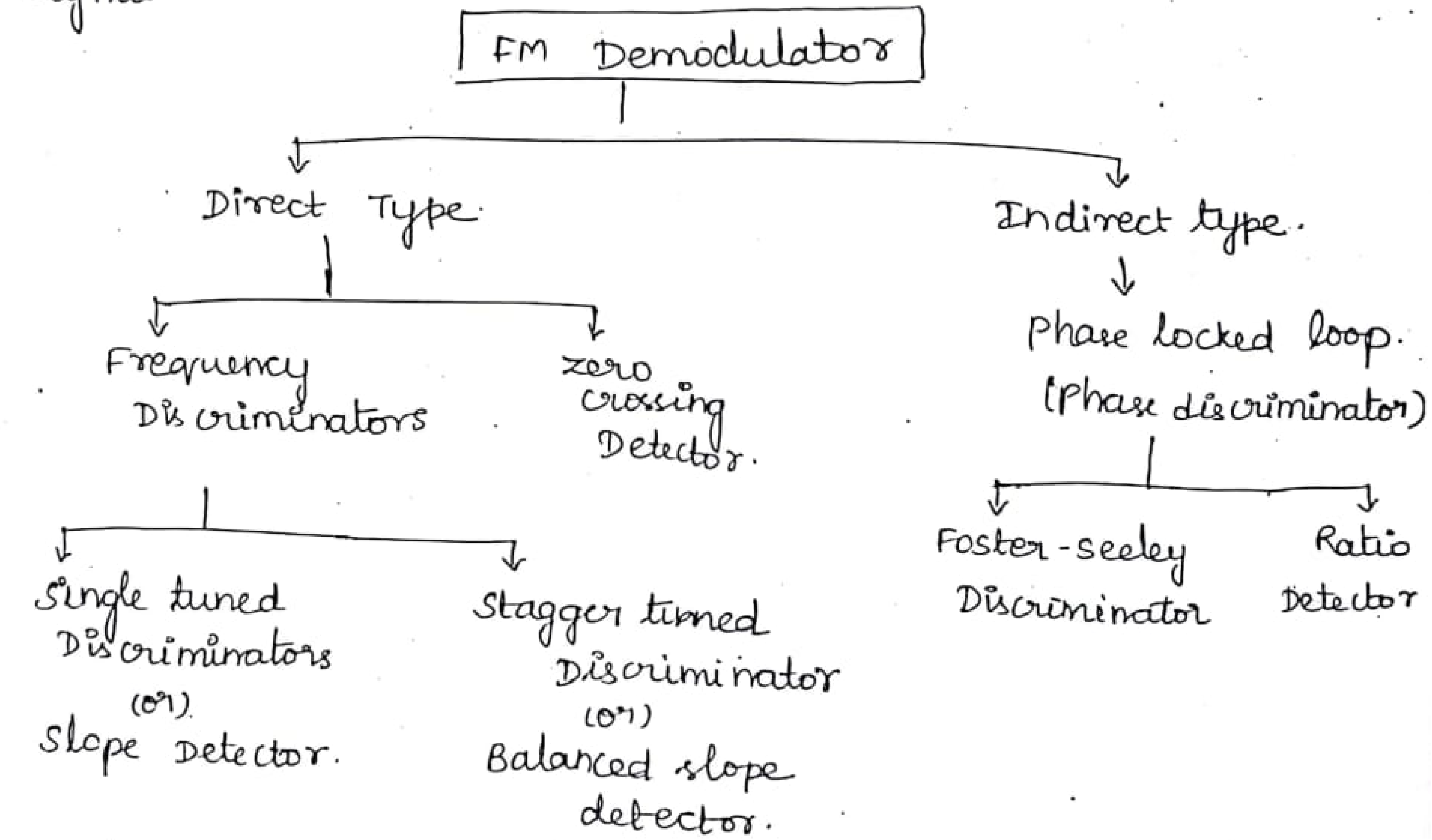
The Eq ③ represents a narrow band FM. Thus at the output of the phase modulator produces a narrow band FM.

Part-II use of Frequency Multipliers & Mixer :-

The FM signal produced at the output of phase modulator has a low carrier frequency and low modulation index. Therefore increased to an adequately high value with the help of frequency multipliers and mixer. The power level is raised to the desired level by the amplifier.

Frequency Demodulation

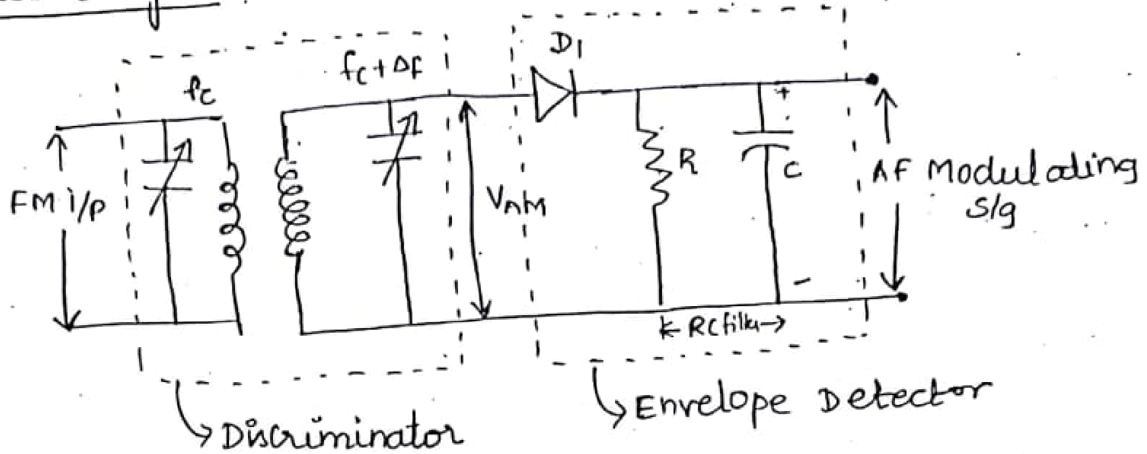
It is exactly opposite to that of frequency modulation. The original message signal is recovered from an incoming FM wave. FM demodulator is basically a frequency to Amplitude converter. It is expected to convert the frequency variations in FM wave at its input into Amplitude variation at its output to recover the original modulating signal.



Slope Detector :-

This detector depends on slope of frequency response characteristics of a frequency selective Network.

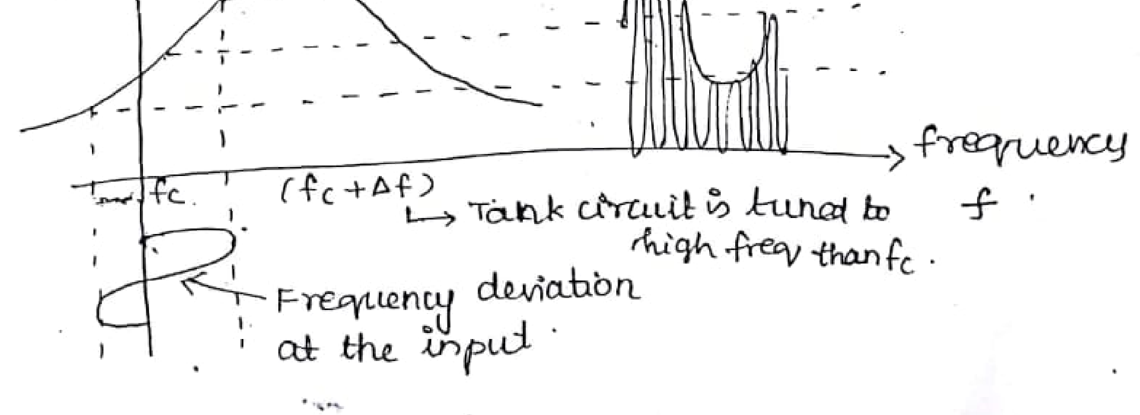
Circuit Diagram :-



The output voltage of the tank circuit is then applied to a simple diode detector of an RC load with proper time constant. This detector is identical to the AM diode Detector.

The circuit is tuned so that its resonant frequency fo is lower than carrier frequency. When the signal frequency increases above fc, the amplitude of the carrier voltage drops. When the signal frequency decreases below fc, the carrier voltage raises.

The change of voltage results because of change in the magnitude of the impedance in the tuned circuit as a function of frequency and results in an effective conversion of frequency modulation into Amplitude modulation. The modulation is recovered from the amplitude modulation using envelope detection.



Drawbacks of slope detector:-

(i) It is inefficient

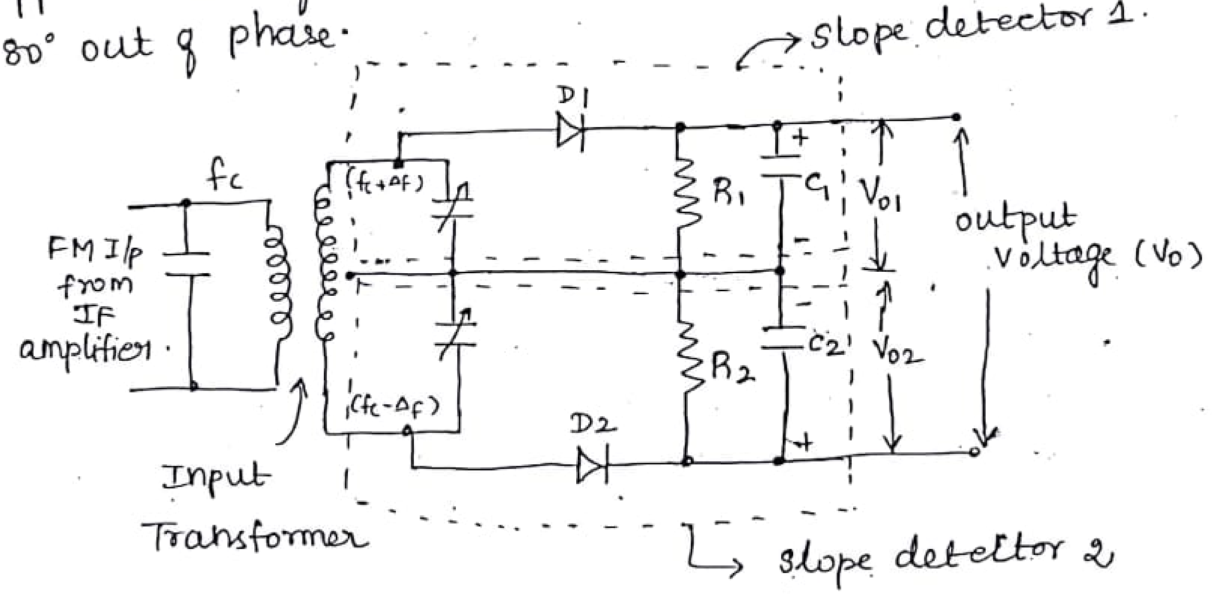
(ii) It is linear only over a limited frequency range.

(iii) It is difficult to adjust as primary & secondary winding of the transformer tuned to slightly different frequencies.

(iv) It does not eliminate the amplitude variations and the o/p is sensitive to any amplitude variations in the input FM signal.

Balanced slope Detector:-

It is used for extending the linearity. It has two slope detector connected back to back in the opposite ends of a center tapped transformer whose input is 180° out of phase.



Balanced slope detector consists of two slope detector circuits. The input transformer has a center tapped secondary. Hence, the input voltage to the two slope detectors are 180° out of phase. There are three tuned circuits.

(i) Primary is tuned to IF ie., fc.

(ii) Secondary upper ckt tuned above fc ie., (fc + Δf)

(iii) Secondary lower circuit tuned below fc ie., (fc - Δf)

∴ R_1C_1 and R_2C_2 are the filters used to bypass the RF ripple. V_{01} and V_{02} are the output Voltages q the two Slope detectors. The final output Voltage V_0 is obtained by taking the subtraction q individual output Voltages.

ie., $\boxed{V_0 = V_{01} - V_{02}}$

working operation q the circuit :-

The circuit operated in three ranges by dividing input frequency.

Case (i) :- $\boxed{f_{in} = f_c}$.

when input frequency f_{in} is ~~eq~~ instantaneously equal to f_c, the induced voltage in T_1 winding q the secondary is exactly equal to that induced in the winding T_2. Thus the input voltages to both diodes D_1 and D_2 will be same. hence Voltages V_{01} & V_{02} will be identical but have opposite polarities $[V_{01} = -V_{02}]$ $\boxed{V_0 = V_{01} + V_{02}}$

∴ $\boxed{V_0 = V_{01} - V_{02} = 0}$

Case (ii) :- $\boxed{f_c < f_{in} < (f_c + \Delta f)}$

Induced voltage in the winding T_1 is higher than the induced in T_2. ∴ D_1 is higher than D_2. Hence positive Voltage V_{01} q D_1 is higher than negative output V_{02} q D_2 ∴ output voltage V_0 is positive.

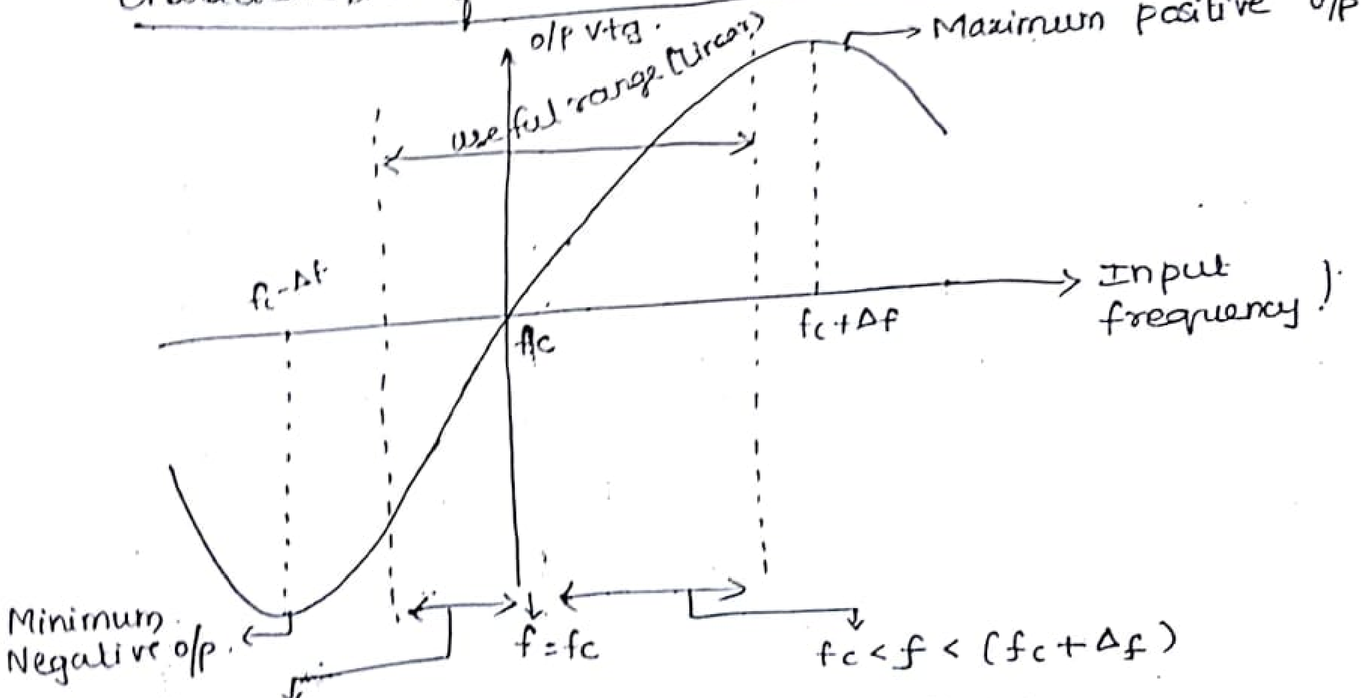
$\boxed{V_0 = \text{positive}}$

case iii :- $\boxed{(f_c - \Delta f) < f_{in} < f_c}$

Induced voltage in winding T_2 is higher than T_1. Input voltage to Diode D_2 is higher than D_1. Hence negative output V_{o2} is greater than V_{o1}. hence output voltage is negative $\boxed{V_o = \text{negative}}$

If the output frequency goes outside the range of $(f_c - \Delta f)$ to $(f_c + \Delta f)$. the output voltage will fall due to reduction in tuned circuit response.

characteristics of Balanced slope detector.



$(f_c - \Delta f) < f < f_c$

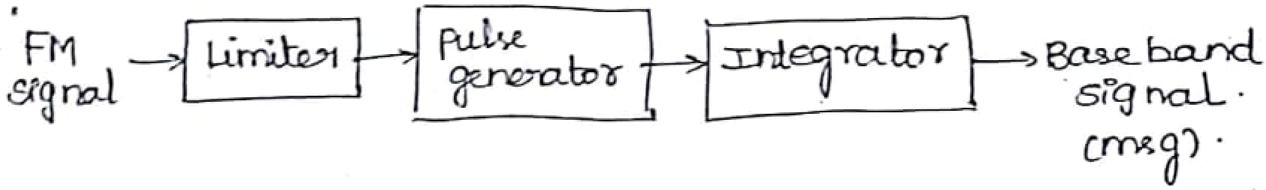
I/p => $D_1 < D_2$
↓
$V_{01} < V_{02}$
↓
$\boxed{V_o \text{ is negative}}$

$D_1 = D_2$
↓
$V_{01} = V_{02}$
↓
$\boxed{V_o = 0}$

I/p => $D_1 > D_2$
↓
$V_{01} > V_{02}$
↓
$\boxed{V_o \text{ is positive}}$

'S'- shape characteristics.

Zero crossing Detector:-



The zerocrossing Detector operated on the principle that the instantaneous frequency of an FM wave.

It is given by $f_i \simeq \frac{1}{2\Delta t}$.

where Δt is time difference between adjacent zero cross over points of the FM wave. The time duration T is chosen, if it satisfies the following two conditions

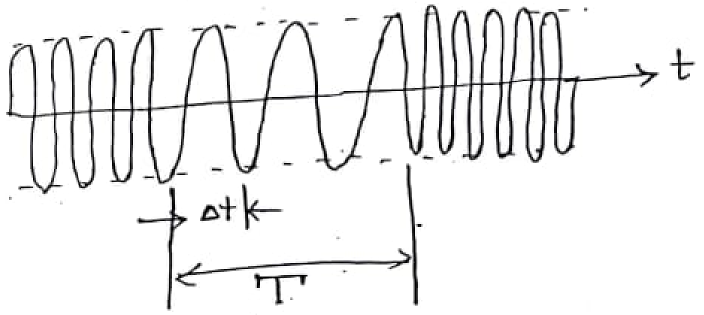
(i) T should be small compared to $(1/W)$ where W is Bandwidth ie., $\boxed{T < 1/W}$

(ii) T should be large compared to $(1/f_c)$ where f_c is carrier frequency.

$$\Delta t = \frac{T}{n_0} \quad \begin{matrix} \rightarrow \text{Time duration.} \\ \rightarrow \text{no. of zero crossings} \end{matrix}$$

Instantaneous frequency $= f_i = \frac{1}{2\Delta t} = \frac{n_0}{2T}$.

There is a linear relation between f_i and message signal $x(t)$. Hence we can recover $x(t)$ if n_0 is known.



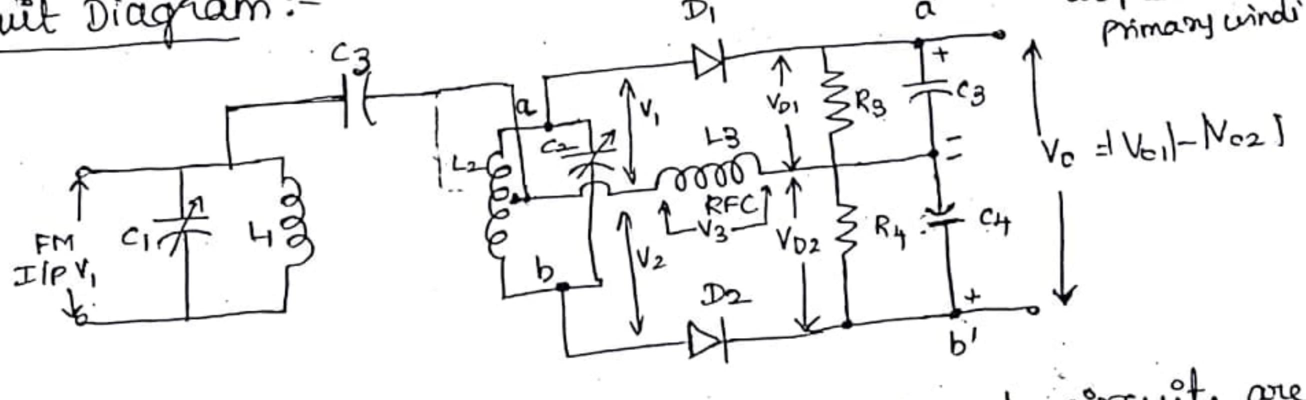
Phase Discriminator [Foster Seeley Discriminator]

Foster seeley discriminator is derived from the balanced modulator(slope detector). because the diode and load arrangement is same as balanced slope detector. but method of applying the input voltage to the diodes which is proportional to the frequency deviation is entirely different.

Foster seeley Discriminator is very sensitive to input amplitude variations and therefore must be preceeded by a limiter. The primary & secondary windings both are tuned to same center frequency fc of the incoming signal. hence it will yield better linearity.

V_3 = primary voltage because it is coupled with primary windi

Circuit Diagram :-



The primary and secondary tuned circuits are tuned to same center frequency, the voltages are applied to two Diodes D_1 and D_2 are not constant. This is due to change in phase shift between primary and secondary windings depending on input frequency.

The current flowing in primary winding of T_1 induces a voltage in secondary winding. Because secondary winding is centre tapped. Voltage across upper portion will be 180° out of phase with voltage across lower portion. Voltage induced in secondary winding is 90° out of phase with voltage across primary.

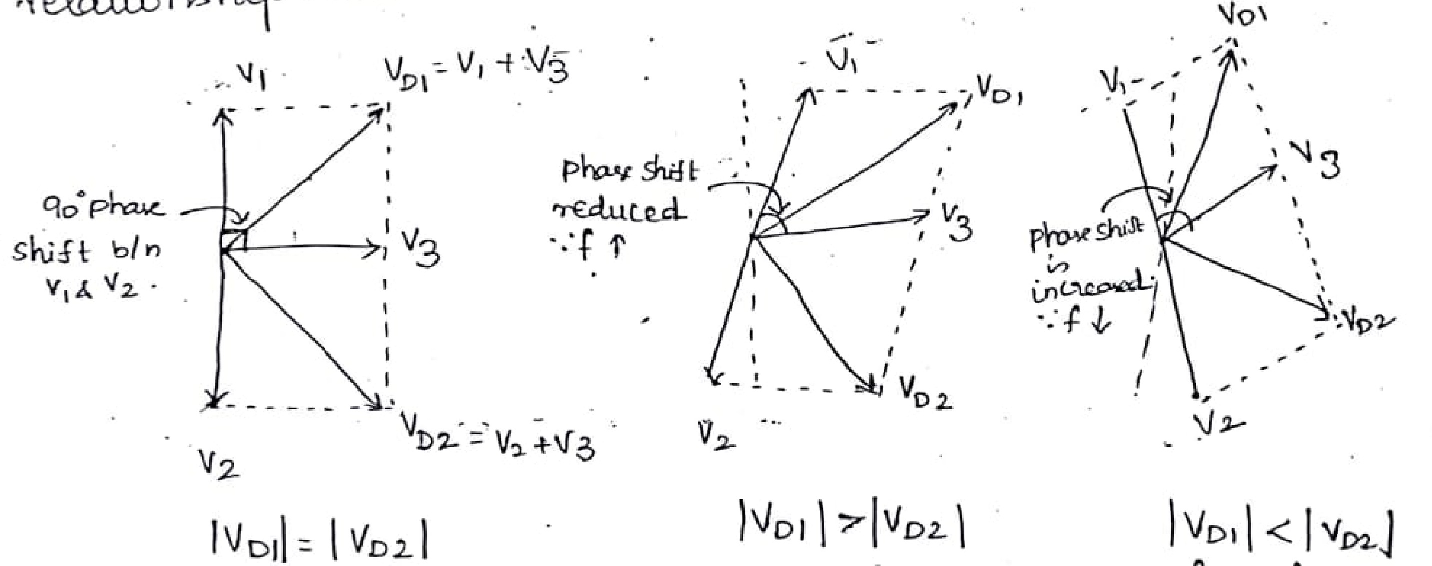
The result is as follows, At resonant Inductive reactance of secondary winding equals capacitive reactance of C2.

(i) At $\boxed{f_{in} = f_c}$, the individual two o/p vtg of Diodes will be equal & opposite. hence $V_o = 0$.

(ii) At $\boxed{f_{in} > f_c}$, The phase shift between the primary and secondary windings is such that output of D1 is higher than D2. hence V_o is positive. circuit becomes Inductive. and V_1 leads V_3 less than 90°; V_2 lags V_3 more than 90°.

(iii) At $\boxed{f_{in} < f_c}$, The phase shift between primary and secondary windings is such that output of D2 is higher than D1, hence V_o is negative. circuit becomes capacitive. V_1 leads V_3 more than 90°; V_2 lags V_3 less than 90°.

The o/p is dependent on primary-secondary phase relationship. hence this circuit is called "Phase Discriminator"



$|V_{D1}| = |V_{D2}|$ $f_{in} = f_c$

$|V_{D1}| > |V_{D2}|$ $f_{in} > f_c$

$|V_{D1}| < |V_{D2}|$ $f_{in} < f_c$.

Drawbacks :-

It does not provide amplitude limiting. so in the presence of noise or any other Spurious Amplitude variations, the demodulator output responds to them and produces errors.

Ratio Detector :-

Ratio Detector is another frequency lemodulator circuit. A primary Advantage of the ratio tetector is that no limiter is needed.

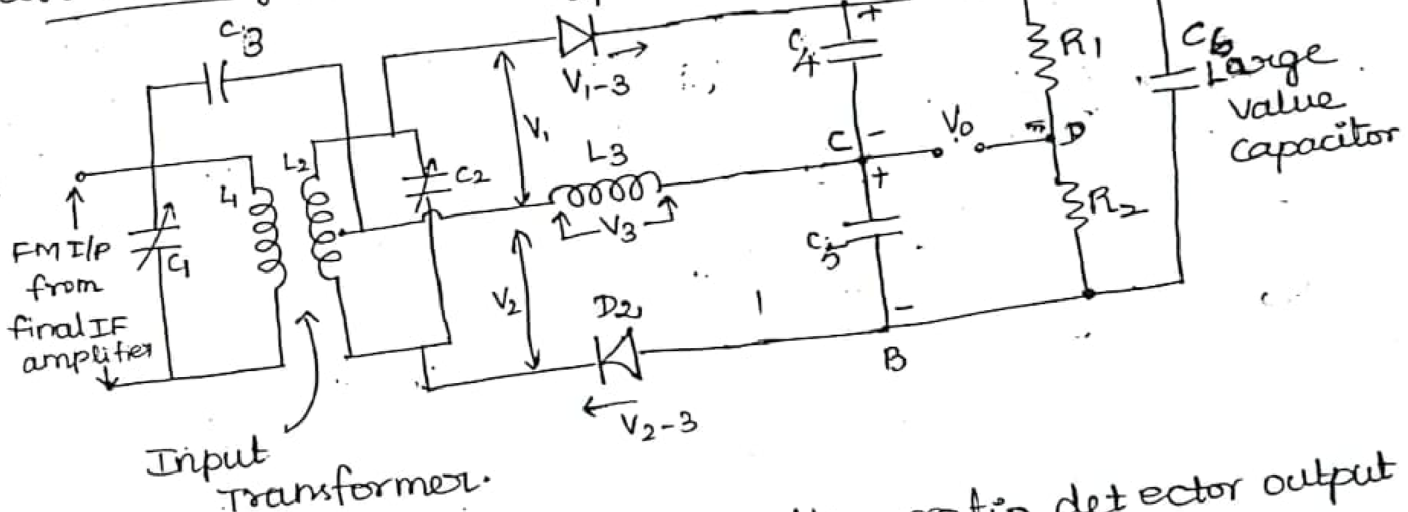
The circuit diagram is similar to Foster seeley discriminator except the following changes.

(i) Direction of Diode D_2 is reversed.

(ii) A large value capacitor C_6 has been included in the circuit. capacitor is made up of Tantalum or electrolytic.

(iii) the output is taken somewhere else.

Circuit diagram :-



It can be shown that the ratio detector output voltage is equal to half of the difference between the output voltages from the individual diodes. Hence, similar to Foster seeley, the output voltage is proportional to the difference between individual o/p voltages. Due to this reason, the operation of ratio detector is identical to the phase discriminator.

DESCRIPTION :-

The load Resistor R_1 & R_2 are equal in value and their common connection is at ground. The output is taken between point C and ground in the circuit. Capacitor C_4 and C_5 & Resistor R_1 & R_2 form a bridge circuit. The voltage across C_4 and C_5 is bridge input voltage, while output is taken between points C and D.

with no modulation on the carrier, the voltage V_{1-3} is applied to D_1 is same as Voltage V_{2-3} applied to D_2. ∴ capacitors C_4 and C_5 charge to same voltage with polarity shown in the circuit. since C_6 is connected across these two capacitors, it will charge to sum of the voltage. It is very large capacitor since it takes several cycles of input signal for the capacitor to charge fully. However once it charges, it will maintain a relatively constant voltage.

since R_1 & R_2 are equal, their voltage drops will be equal. Also voltage Drops C_4 & C_5 are equal. The bridge circuit is therefore balanced. ∴ Between the points C & D, potential is same hence 0V. Assume at center carrier frequency, the voltage Drops across C_4 & C_5 are each 2V. This means the charge on C_6 is 4V. Then voltage across R_1 & R_2 = 2V each.

If frequency increases, the phase relationship in the circuit will change. this will cause the voltage across C_4 to be greater than voltage across C_5. Assume that voltage across C_4 = 3V & C_5 = 1V. but the voltage across R_1 & R_2 remain same at 2V each. Because charge on C_6 does not change. The bridge is now unbalanced.

An output voltage will appear between points C & D in the circuit. using point B as reference, the voltage at point C is 1V positive and voltage across R2 is 2V positive.
∴ voltage Difference at C is -1V.

If the frequency decreases, then the phase relationship will be such that the charge on C5 will be greater than charge on C4. If the voltage across C5 is +3V. with respect to B. and voltage across R2 remains 2 V. then at point C is +1V. the bridge is unbalanced but in opposite direction, and the o/p vtg is of opposite polarity.

The primary advantage of ratio detector over discriminator is that essentially insensitive to noise and amplitude variations. The C6 (very large capacitor) takes long time to charge or discharge. Shot noise pulses and minor amplitude variation are totally smoothed out.

However, the average DC voltage across C6 is same as average signal amplitude. This voltage can therefore be used in automatic gain control applications.

The ratio detector and Foster seeley discriminator are no longer widely used. because they are difficult to implement in integrated circuit form. Besides, the Quadrature demodulator and PLL offer far superior performance for comparable cost.

Operation :-

The polarity of V_{o2} is reversed, since connections of D2 are reversed. hence the voltages V_{o1} and V_{o2} across two capacitors add (Note that two voltages subtract in Foster seeley circuit). when V_{o1} increases, V_{o2} decreases and viceversa.

o/p vtg due to Diode D1 :-

$$V_o = V_{o1} - \frac{V_R}{2} \qquad \left[\text{But } V_R = V_{o1} + V_{o2}\right]$$

$$V_o = V_{o1} - \left[\frac{V_{o1}+V_{o2}}{2}\right]$$

$$\boxed{V_o = \frac{V_{o1}-V_{o2}}{2}} \longrightarrow ①$$

o/p vtg due to Diode D2 :-

$$V_o = -V_{o2} + \frac{V_R}{2} \qquad \left[\text{But } V_R = V_{o1} + V_{o2}\right]$$

$$V_o = -V_{o2} + \left(\frac{V_{o1}+V_{o2}}{2}\right)$$

$$\boxed{V_o = \frac{V_{o1}-V_{o2}}{2}} \longrightarrow ②$$

o/p vtg of ratio Detector :-

Adding ① & ② we get,

$$2V_o = \left(\frac{V_{o1}-V_{o2}}{2}\right) + \left(\frac{-V_{o2}+V_{o1}}{2}\right) = (V_{o1}-V_{o2})$$

$$V_o = \frac{1}{2}(V_{o1}-V_{o2})$$

$$\simeq \frac{1}{2}(|V_{D1}|-|V_{D2}|) \longrightarrow ③$$

The output of ratio detector is half compared to that of Foster seeley circuit.

Merits :-

(i) Easy to align.

(ii) Very good linearity.

(iii) Amplitude limiting is provided inherently.

(iv) It has reduced fluctuations in the output voltage.

Demerits :-

The ratio detector may not tolerate the long period variation in signal strength. This requires an AGC signal.

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PLL as FM Demodulator

A Phase locked loop (PLL) is primarily used in tracking the phase and Frequency of the carrier component of an incoming FM signal. PLL is also useful for synchronous demodulation of AM-SC. and FM signals in presence of large noise and low signal power.

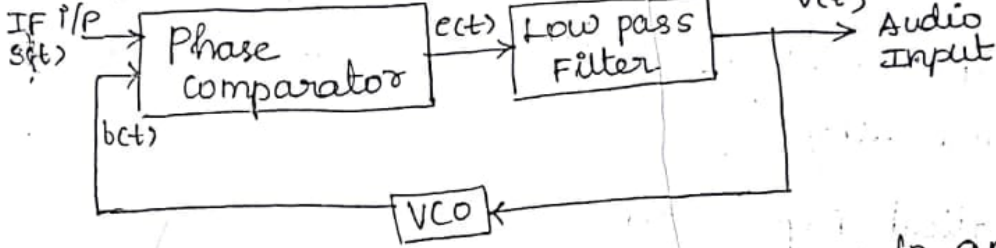
Hence PLL is most suitable for use in space vehicle to earth data links or where the loss along the transmission line or path is Quite large.

A PLL is basically a negative Feedback system. It consists of three major components. These components are multiplier, loop filter and a voltage controlled oscillator connected together in the form of feedback loop.

A VCO [Voltage controlled oscillator] is a sine wave generator whose frequency is determined by the voltage applied to it from an external source.

Initially adjust the VCO so that when control vtg is 0.
Two conditions are satisfied,
(i) frequency of VCO is precisely set at unmodulated f_c.
(ii) VCO o/p has 90° phase shift w.r.t unmodulated carrier.

Block diagram :-



A phase comparator compares the VCO o/p and the IF input signal. When there is no modulation, the received IF frequency and VCO oscillator frequency are exactly same and phase comparator circuits puts out a zero signal.

When the incoming frequency changes due to presence of modulation, or frequency deviation. The phase comparator creates and output which drive the VCO frequency up or down until it again matches the incoming IF. Thus the PLL tracks the incoming IF signal.

The signal appearing at the i/p to VCO is the sum of fixed dc bias plus the comparator output signal. Every time VCO oscillator frequency changes according to the deviation present in FM signal and the value of voltage of VCO i/p will vary about the bias value in accordance with the modulating signal.

A low pass filter will remove the carrier components and dc components are filtered leaving only the modulating signal.

A PLL can track the incoming frequency only over a finite range of frequency shift. This range is called "lock range or holdin range". Lock range is different for various types of PLL.

Also if the i/p frequency changes too rapidly, the loop may not lock. The frequency range over which the input will cause the loop to lock is called "pull-in or capture range."

Mathematical Expression:-

$$s(t) = A \sin[\omega_c t + \phi_1(t)] \; ; \; b(t) = A_V \cos[\omega_c t + \phi_2(t)] .$$

where $A \rightarrow$ Amplitude of unmodulated carrier.
$A_V \rightarrow$ Amplitude of VCO o/p.

$$\boxed{\phi_1(t) = 2\pi K_f \int_0^t x(t)\,dt}$$ $K_f \rightarrow$ freq. sensitivity of FM

$$\boxed{\phi_2(t) = 2\pi K_v \int_0^t v(t)\,dt .}$$ $K_v \rightarrow$ freq. sensitivity of VCO

o/p of phase comparator,

$$e(t) = s(t) \cdot b(t) .$$

$$= A A_V \sin[\omega_c t + \phi_1(t)] \cos[\omega_c t + \phi_2(t)]$$

$$e(t) = \frac{A A_V}{2} \left[\sin(2\omega_c t + \phi_1(t) + \phi_2(t)) \; \sin(\phi_1(t) - \phi_2(t)) \right].$$

It is pass on to LPF, hence it neglect high freq. terms.

$$e(t) = K_m \frac{A A_V}{2} \sin(\phi_1(t) - \phi_2(t))$$

$$e(t) = K_m \frac{A A_V}{2} \sin(\phi_e(t))$$

where $K_m \rightarrow$ Multiplier gain measured in per volt.
$\phi_e(t) \rightarrow \phi_1(t) - \phi_2(t)$ [phase error].

$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$

$$= \phi_1(t) - 2\pi K_v \int_0^t v(t)\,dt\,.$$

The loop filter operates on error signal e(t) to produce output v(t).

$$v(t) = e(t) * h(t).$$

$$\therefore v(t) = \int_{-\infty}^{\infty} e(\tau) h(t-\tau)\,d\tau.$$

$$\therefore \phi_e(t) = \phi_1(t) - 2\pi k_v \int_0^t \int_{-\infty}^{\infty} e(\tau) h(t-\tau)\,d\tau\,dt.$$

$$\phi_e(t) = \phi_1(t) - 2\pi k_v \int_0^t \int_{-\infty}^{\infty} \sin[\phi_e(\tau)]\, h(t-\tau)\,d\tau\,dt$$

Differentiating on both sides,

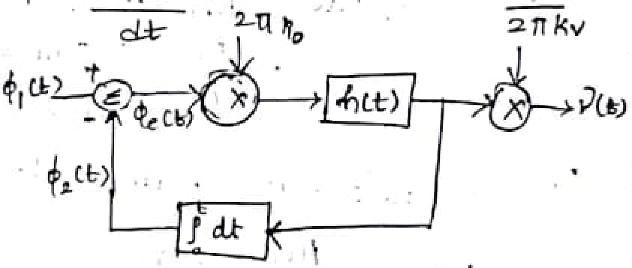
$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi k_v \left[\int_{-\infty}^{\infty} \sin\phi_e(\tau)\, h(t-\tau)\,d\tau\right]$$

$$[\sin\phi_e(\tau) \simeq \phi_e(\tau)]$$

$$\frac{d\phi_e(t)}{dt} + 2\pi k_v \int_{-\infty}^{\infty} \phi_e(\tau) h(t-\tau)\,d\tau = \frac{d\phi_1(t)}{dt}$$

Apply F.T on both sides,

$$\boxed{\phi_e(f) = \frac{1}{1 + k_v \frac{H(f)}{jf}}\,\phi_1(f).}$$



linearized model q PLL.

where $\left(k_v \frac{H(f)}{jf}\right)$ is called open loop transfer function q PLL.

——x——

Transmission Bandwidth :-

The effective bandwidth is defined as the difference between the two extreme significant sideband frequencies on either side of FM signal.

$$\boxed{B\cdot W = f_H - f_L}$$

There are many ways to find out transmission Bandwidth of FM. They are

(i) universal curve

(ii) carson's rule

(iii) Bessel Table.

universal curve :-

It is defined as the separation between the two frequencies beyond which none of the side frequencies is greater than 1% of carrier Amplitude obtained when the modulation is removed.

$$\boxed{B_T = 2Nf_m}$$

where $B_T \rightarrow$ Transmission Bandwidth

$N \rightarrow$ significant sidebands.

$f_m \rightarrow$ modulating frequency.

Carson's rule :- (or) Thumb rule.

According to carson, B.W of FM signal is equal to twice the sum of frequency deviation and maximum modulating frequency. $\boxed{B_T = 2(\Delta f + f_m)} = 2f_m(\beta + 1)$.

Deviation ratio is defined as ratio of maximum freq deviation to B.w of modulating signal. It is similar to modulation Index(β) in a single tone FM.

$$\boxed{DR = \Delta f / W}$$

$$\boxed{B_T = 2f_m(DR+1)} \qquad (\because \beta = DR)$$

— × —

Bandwidth of FM using Bessel Table :-

99% of Bandwidth of FM wave as the Separation between two frequencies beyond which none of the side frequencies is greater than 1% of Carrier amplitude obtained when modulation is removed.

$$B.W = 2 n_{max} f_m$$

f_m → modulating frequency

n_{max} → maximum value of integer n.

n_{max} Satisfies the requirement $|J_n(\beta)| > 0.01$. It varies with β.

S.no	modulation index β	no of significant sidebands $2n_{max}$
1	0.1	2
2	0.3	4.
3	0.5	4
4	1.0	6
5	2.0	8
6	5.0	16
7	10.0	28
8	20.0	50
9	30.0	70.

———X———

UNIT - III

Random Process.

Random variables :

A function which takes on any value from the sample space and its range is some set of real numbers is called a random variable of the experiment.

A random variable is not random since it takes values from well defined sample space. It is not variable since it has fixed value when it occurs.

If the outcome of the experiment is the sample point 's', then then the random variable is represented as $x(s)$.

ex :

If we toss a coin, the possible outcomes are Head (H) and Tail (T).

$\therefore$ The sample space contains two sample points.

$$S = \{H, T\}.$$

If we define the function x such that

$$x = \begin{cases} 1 & \text{when } S = H \\ -1 & \text{when } S = T \end{cases} \quad \text{then } x = \{-1, 1\}.$$

Consider another experiment of throwing a die. The sample space for this experiment consist of six possible outcomes.

(ie) $S = \{1, 2, 3, 4, 5, 6\}$.

If we define random variable as $X = S^2$

then $X = \{1, 4, 9, 16, 25, 36\}$.

Types of Random variables..

1. Discrete Random variable.
2. Continuous Random variable.

Discrete Random variable..

The random variable X is a discrete random variable if X can take on only finite number of values in any finite observation interval. Thus the discrete random variable has countable number of distinct values.

ex

$X = \{1, 4, 9, 16, 25, 36\}$

is discrete random variable.

Continuous Random variables :

There are many physical systems that generate continuous outputs (outcomes). Such systems generate infinite number of outputs (outcomes) within the finite period. continuous random variables can be used to define the outputs of such systems.

If the random variable 'x' takes on any value in a whole observation interval, x is called continuous random variable.

ex :

The noise voltage generated by an electronic amplifier has a continuous amplitude. Therefore sample space S of the noise voltage amplitude is continuous ~~amplitude~~. ∴ The random variable x has continuous range of values. The random variable takes uncountable number of possible values.

Cumulative Distribution Function. (CDF).

Cumulative Distribution Function provides probabilistic description of a random variable.

The cumulative Distribution function (CDF) of a random variable 'X' is the probability that a random variable 'x' takes a value less than or equal to x.

x is the dummy variable.

Let us consider the probability of the event $X \leq x$. The probability of this event can be denoted as $P(X \leq x)$. Then from definition of cumulative distribution function,

$$F_X(x) = P(X \leq x).$$

$F_X(x)$ is called cumulative Distribution function of random variable 'X'.

Properties of CDF :

Property 1 :

The CDF is bounded between 0 and 1. (ie) $0 \leq F_X(x) \leq 1$

Property 2 :

$F_X(-\infty) = 0$ and $F_X(\infty) = 1$

$x = -\infty$ means no possible event

Hence $P(X \leq -\infty) = 0$

At $x = \infty$ means $P(X \leq \infty)$ includes probability of all possible events

$\therefore P(X \leq \infty) = 1.$

Property 3 :

$F_X(x_1) \leq F_X(x_2)$. if $x_1 \leq x_2$.

"Numerical characteristics:

- It includes characteristics of position and characteristics of dispersion.

Characteristics of position:

a) Mean or expected value:

$$\bar{x} = \mu' = E(x) = \begin{cases} \sum_{j} x_j p_j, & \text{if } x \text{ is a discrete variable} \\ \int_{-\infty}^{\infty} x f_x(x)\,dx, & \text{if } x \text{ is a continuous variable} \end{cases}$$

b) Mode:

The mode of a continuous variable x is a real number d_x defined to be the maximum point of the probability density $f_x(x)$.

$$P(x = x_m) = \max_{K} P(x = x_K)$$

c) Median:

$$P(x \le h_x) = P(x \ge h_x)$$ $h_x \to$ root of the equation

Characteristics of Dispersion:

a) Variance:

The variance of a random variable x is a non-negative number.

$$Var(x) = \mu_2 = E((x-\bar{x})^2) = \begin{cases} \sum_{K} (x_K - \bar{x})^2 P_K, & \text{if } x \text{ is a discrete variable} \\ \int_{-\infty}^{\infty} (x-\bar{x})^2 f_x(x)\,dx, & \text{if } x \text{ is a continuous variable} \end{cases}$$

b) standard deviation (or) mean square deviation:

$$\sigma_x = \sqrt{Var(x)}$$

c) Raw moments:

$$\mu'_m = E(x^m) = \begin{cases} \sum_{K} x_K^m P_K, & \text{if } x \text{ is a discrete} \\ \int_{-\infty}^{\infty} x^m f_x(x)\,dx, & \text{if } x \text{ is a continuous variable} \end{cases}$$

d) Central moment (or) moments about mean:

$$\mu_m = E((x - \bar{x})^m) = \begin{cases} \sum_k (x_k - \bar{x})^m p_k, & x \text{ is a discrete} \\ \int_{-\infty}^{\infty} (x - \bar{x})^m f_x(x)\,dx, & x \text{ is a continuous variable.} \end{cases}$$

Distributions:

a) Discrete distributions:

a) Binomial distribution:

$$P(X = k) = \binom{n}{k} p^k q^{n-k} \quad \text{for } k = 1, 2 \ldots n$$

$q = 1 - p$.

b) Poisson distribution:

$$P(X = k) = \frac{e^{-\lambda} \lambda^k}{k!} \quad \text{for } k = 0, 1, 2 \ldots$$

λ → positive real number.

Continuous distributions:

a) Uniform distribution:

$$F(x) = \begin{cases} 0, & \text{if } x \leq a \\ \frac{x - a}{b - a}, & \text{if } a < x < b \\ 1, & \text{if } x \geq b \end{cases}$$

b) Cauchy's distribution:

$$F(x) = \frac{1}{2} + \frac{1}{\pi} \arctan x, \quad f(x) = \frac{1}{\pi(1 + x^2)}$$

c) Exponential distribution:

$$F(x) = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Density function

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

d) Normal distribution:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x - m}{\sigma}\right)^2 / 2}$$

$m \rightarrow$ mean,

$\sigma^2 \rightarrow$ Variance.

Random Process:

* The Random process x(t) is defined as an ensemble of time functions together with a probability rule that assigns a probability to any meaningful event associated with an observation of one of the sample functions of the random process.

* A random process defined as a function of one or more random variables as,

$$x(t) = g(y_1, y_2 \dots y_n ; t)$$

Where, x(t) → random process

$y_1, y_2 \dots y_n$ → 'n' random variable

g → ordinary function

Classifications:-

1. A process is said to be continuous random process if x and t are continuous.
2. A process is said to be continuous random sequence (discrete time continuous state space) if x is continuous and t is discrete
3. A process is said to be Discrete random process (Continuous time discrete state space) if x is discrete and t is continuous.
4. A process is said to be discrete random sequence (Discrete time discrete state space). If x and t are discrete.

Deterministic random variable:

* If all the feature value can be predicted from past observations. ie. if x(t, s) is known for $t \leq t_0$ then it is determined for $t > t_0$, then the process is called deterministic.

Non-deterministic random variable:

* If future values of any sample function cannot be predicted from past observations.

Statistics of Random process:

A random process is a collection of infinite number of random variables for each fixed 't'. Thus for a specific 't', $x(t)$ is a random variable with distribution function $F(x,t) = P(x(t) \leq x)$

a) First order distribution of the process $x(t)$:

* The distribution function $F(x,t)$ of a process $x(t)$ at a specific time t. will be called the first order distribution of the process $x(t)$.

$$f(x,t) = \frac{\partial F(x,t)}{\partial x}$$

b) Second order distribution:

$$F(x, x_2 : t_1, t_2) = P(x(t_1) \leq x_1, x(t_2) \leq x_2)$$

Statistical Averages

i) Mean:

The mean of the process $x(t)$ is the expected value of the random variable x at time t.

$$\mu = E(x(t)) = \int_{-\infty}^{\infty} x \, f(x,t) \, dt$$

ii) Autocorrelation:

The Autocorrelation $R(t_1, t_2)$ of the random process $x(t)$ is the expected value of the product $x(t_1)x(t_2)$

$$R(t_1, t_2) = E(x(t_1)x(t_2)) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 \, f(x_1, x_2 : t_1', t_2) \, dx_1 \, dx_2$$

iii) Average power of $x(t)$:

$$E(x^2(t)) = R(t,t) \qquad t_1 = t_2 = t$$

iv) Auto covariance:

$$C(t_1, t_2) = R(t_1, t_2) - E(x(t_1))E(x(t_2))$$

if $t_1 = t_2$

$$C(t_1, t_1) = Var(x(t))$$

Correlation coefficient:

$$r_{xx}(t_1, t_2) = \frac{C(t_1, t_2)}{\sqrt{C(t_1, t_1)\, C(t_2, t_2)}}$$

if $t_1 = t_2 = t \Rightarrow r_{xx}(t, t) = 1$

Cross correlation:

The cross correlation function of two random process is defined as a measure of the similarity between a signal and a time delayed version of a second signal.

$$R_{xy}(t_1, t_2) = E(x(t_1)\, y(t_2)) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f(x, y : t_1, t_2)\, dx\, dy$$

Cross covariance:

$$C_{xy}(t_1, t_2) = R_{xy}(t_1, t_2) - E(x(t_1))\, E(y(t_2))$$

Cross correlation coefficient:

$$r_{xy}(t_1, t_2) = \frac{C_{xy}(t_1, t_2)}{\sqrt{C_{xx}(t_1, t_1)\, C_{yy}(t_2, t_2)}}$$

Orthogonal Process:

$$R_{xy}(t_1, t_2) = 0, \forall t_1 \text{ \& } t_2$$

Uncorrelated process:

$C_{xy}(t_1, t_2) = 0$. for every t_1 and t_2

Time Averages:

If $x(t)$ is a random process, then

$\overline{x_T} = \frac{1}{2T}\int_{-T}^{T} x(t)\, dt$ is called time average of $x(t)$ over $(-T, T)$.

Stationary Process:

a) Strict Sense Stationary:

A process $x(t)$ is called SSS if its statistical properties are invariant to a shift of the origin. ie the processes $x(t)$ and $x(t+c)$ have the same statistics for any c.

First order stationary process:

A process is called stationary to first order, if its first order density function does not change with a shift in the time origin. ie $f_x(x_1;t_1) = f_x(x_1;t_1+c)$

Second order stationary process:

A process is called stationary to order two, if its second order density function does not change with shift in the time.

$$f_x(x_1, x_2; t_1, t_2) = f_x(x_1, x_2; t_1+\tau, t_2+c)$$

$\forall$ values of t_1, t_2 and c

Wide Sense Stationary process:

A random process $x(t)$ is called WSS if:

i) Its mean is const. ie. $E(x(t)) = \bar{x} = \text{const}$.

ii) Its auto correlation depends only on $\tau = t_1 - t_2$

ie) $E(x(t+\tau)\,x(t)) : R(\tau)$

Ergodic process:

If for a stationary process all the time averages are equal to corresponding statistical average the process is called as ergodic process.

$$\langle x(t) \rangle = E(x(t)) = \bar{x}$$

In general $\langle x^n(t) \rangle = E(x^n(t))$, $n = 1, 2 \ldots$

Mean ergodic:

A WSS is said to be ergodic in the mean if the time averages of $x(t)$ converges to the ensemble average $E(x(t))$

$$\langle x(t) \rangle = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} x^2(t)\,dt \rightarrow E(x(t))$$

Ergodic in mean square:

$$\langle x^2(t)\rangle = \underset{T\to\infty}{Lt}\frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt = R_x(0)$$

Ergodic in correlation:

A WSS x(t) is ergodic in correlation at the time shift τ.

$$\langle x(t+\tau)x(t)\rangle = \underset{T\to\infty}{Lt}\frac{1}{2T}\int_{-T}^{T} x(t+\tau)x(t)\,dt = R_{xx}(\tau)$$

Gaussian Process:

The process x(t) is a Gaussian process if every linear functional of x(t) is a Gaussian random variable.

$$f(x_1,x_2\ldots x_n;t_1,t_2\ldots t_n) = \frac{1}{(2\pi)^{n/2}}\frac{1}{\Delta^{1/2}}\exp\left(\frac{-1}{2\Delta}\sum_{i=1}^{n}\sum_{j=1}^{n}\Delta ij(x_i-\mu_i)(x_j-\mu_j)\right)$$

For the first order density of a Gaussian process

$\Delta = |\lambda_n| = |cov(x(t_1).x(t_1))| = |Var(x(t_1))| = \sigma_1^2$ and $\Delta_{11} = 1$

$$\therefore f(x_1;t_1) = \frac{1}{\sigma_1\sqrt{2\pi}}\exp\left(-(x_1-\mu_1)^2/2\sigma^2\right)$$

For the second order density, $r_{12} = r_{21} = r$

$$f(x_1,x_2;t_1,t_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-r^2}}\exp\left(\frac{-1}{2\sqrt{1-r^2}}\left(\frac{(x_1-\mu_1)^2}{\sigma_1^2} - 2r\frac{(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2} + \frac{(x_2-\mu_2)^2}{\sigma_2^2}\right)\right)$$

Properties:

1. If a Gaussian process x(t) is applied to a stable filter, then the random process y(t) developed at the output of the filter is also Gaussian.
2. If the random variable $x(t_1)\ldots x(t_n)$ obtained by sampling a Gaussian process x(t) at time $t_1, t_2\ldots t_n$ are uncorrelated

   ie) $E\left(x(t_i)-\mu_x(t_i)\right)\left(x(t_j)-\mu_x(t_j)\right) = 0, i\neq j$

   then these random variables are statistically independent.
3. If a Gaussian process is stationary, then the process is also strictly stationary.

Transmission of a Random Process through LTI filter: (APR/MAY 2018) (NOV/DEC 2016) (MAY/JUNE 2016)

Consider LTI filter with impulse response h(t). A random process x(t) at input and output random process y(t) are related by Convolution integral,

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau.$$

Mean value of output random process will be,

$$m_y(t) = E\left[y(t)\right]$$

$$= E\left[\int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau\right]$$

Interchanging the order of expectation and integration,

$$m_y(t) = \int_{-\infty}^{\infty} h(\tau)\, E\left[x(t-\tau)\right] d\tau.$$

$$= \int_{-\infty}^{\infty} h(\tau)\, m_x(t-\tau)\, d\tau.$$

$E\left[x(t-\tau)\right] = m_x(t-\tau) \Rightarrow$ mean.

Since $x(t)$ is stationary $m_x(t-\tau) = m_x(t)$

$$m_y(t) = m_x(t) \int_{-\infty}^{\infty} h(\tau)\, d\tau.$$

$$= m_x(t) \cdot H(0).$$

Here $\int_{-\infty}^{\infty} h(\tau) d\tau = H(0)$ is the DC response of the system.

UNIT IV

Noise CHARACTERIZATION

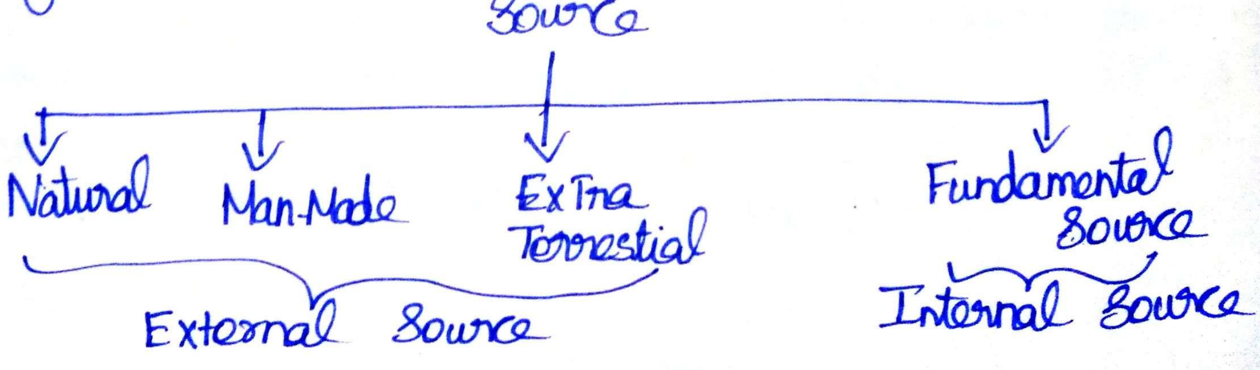
Noise Sources - Noise figure, noise temperature and Noise Bandwidth - Noise in Cascaded Systems. Representation of Narrow Band Noise - In phase and Quadrature, Envelope and phase Noise performance analysis in AM & FM systems - Threshold effect, pre emphasis and de emphasis for FM.

Noise.

It is unwanted signals that tend to disturb the transmission and processing of signals in Communication System and over which we have incomplete Control (or) the spontaneous fluctuations of current or voltage in electrical circuits.

Sources of Noise.

The Noise can arise from different type of sources.



Natural Source Of Noise

The natural phenomena that give rise to noise are electronic storms, solar flares and addition in space. The noise received by receiving antenna from the natural source can only be reduced by repositioning the antenna.

Man-Made Source

It is also called Industrial Noise. The man made Noise is generated due to the make and break process of a current carrying circuit. The examples are electrical motors, welding Machines, ignition System of automobiles, Switching gear, Fluorescent lights etc.

Extra-Terrestial Noise

The Noise originating from Sun and the Space is known as Extra Terrestial Noise. It is subdivided into two groups

(a) Solar Noise - Comes from Sun

(b) Cosmic Noise - Comes from Stars.

Our Sun is being a large body at Very high temperature radiates a lot of Noise.

Our Stars also large & hot bodies. This Cosmic noise is called as black body Noise. and it is uniformly distributed over the entire Sky.

Fundamental Source Of Noise.

This Noise occurs with in the electronic equipment. They are called Fundamental Source because they are integral part of physical nature of the material. It can be eliminated by proper design in electronic circuits & equipments.

Types of Noise

The fundamental noise source produce different types of noise. They are as follows

Types.

- Shot Noise
- Thermal Noise (or) Jhonson Noise
- White Noise (or) Guassian Noise
- Partition Noise
- flicker Noise (or) Pink Noise (or) Low frequency Noise
- Transit time Noise (or) High freq Noise
- Burst error Noise (or) Brown Noise

Noise Figure

When noise factor 'F' is expressed in decibals. It is Known as noise figure.

$$\text{Noise Figure } F_{dB} = 10\log_{10} F$$

$$= 10\log_{10}\left[\frac{S/N \text{ at input}}{S/N \text{ at output}}\right]$$

The Noise factor F of an amplifier or any network is defined interm of Signal to noise ratio at the input and the output of the system. It is defined as

$$F = \frac{S/N \text{ ratio at the input}}{S/N \text{ ratio at the Output}} = \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}} \rightarrow ①$$

where P_{si} & P_{ni} = Signal & noise power at the i/p

P_{so} & P_{no} = Signal & noise power at the o/p

The temperature to calculate the noise power is assumed to be room temperature. The S/N at the input will always be greater than at output. This is due to noise added by the amplifier. Hence, the noise factor is means to measure the amount of noise added and it will be always greater than one. The ideal value of noise factor is unity.

The noise factor F is sometimes frequency dependent. Then its value determined at one frequency is known as spot noise factor and the frequency must be stated along with noise factor.

The available power gain $G = \frac{P_{so}}{P_{si}} \rightarrow ②$

Substitute ② in ①

$$F = \frac{P_{no}}{G P_{ni}} \rightarrow ③$$

Therefore the noise power at the amplifier output is $P_{no} = F G P_{ni} \rightarrow ④$

but $P_{ni} = KTB$ $(\because T = T_0 \text{ room Temp})$

$$P_{no} = F G K T_0 B \rightarrow ⑤$$

Noise Factor interms of R_n.

$$(S/N)_{out} = \frac{V_s^2}{4KT_0B(R_p+R_n)}$$

$R_p \rightarrow$ Parallel combination of amplifier R_i & R_s.

$$(i.e)\ R_p = \frac{R_i R_s}{R_i + R_s}$$

$R_n \rightarrow$ Noise resistance

$$(S/N)_{in} = \frac{V_s^2}{4KT_0BR_p}$$

Hence Noise factor $F = \frac{(S/N)_{in}}{(S/N)_{out}} = \frac{R_p + R_n}{R_p}$

If the amplifier does not produce any noise (i.e) $R_n = 0$, under this condition, noise factor will be unity.

$$\text{Noise figure} = 10\log_{10}\left[\frac{S/N \text{ at the input}}{S/N \text{ at the output}}\right]$$

$$= 10\log_{10}(S/N)_i - 10\log_{10}(S/N)_o$$

$$F_{dB} = (S/N)_i\, dB - (S/N)_o\, dB.$$

The ideal value of Noise figure is 0 dB.

Methods to Improve noise figure

(i) Use diodes & FET for amplifiers and mixer stages.

(ii) Receiver can operate at low temperature

(iii) use high gain Amplifiers

Noise temperature

It is the another way to represent the noise by means of equivalent noise temperature, is used in dealing with UHF and microwave low noise antennas, receivers or devices.

The total Noise referred to the input of amplifier is

$$P_{ni}\,(\text{total}) = \frac{P_{no}}{G} \quad \begin{matrix} \rightarrow \text{Noise power at o/p} \\ \rightarrow \text{Gain of amplifier} \end{matrix}$$

But

$$P_{no} = FGKT_0B$$

$$P_{ni} = KT_0B$$

$$P_{ni}\,(\text{total}) = \frac{FGKT_0B}{G} = FKT_0B$$

Out of this total i/p noise power, the input source contribution is only KT_0B and remaining is contributed by the amplifier

$$P_{ni}(\text{total}) = P_{ni} + P_{na}.$$

$$P_{na} = P_{ni}\,(\text{total}) - P_{ni} = FKT_0B - KT_0B$$

$$P_{na} = (F-1)\,KT_0B$$

$$KT_{eq}B = (F-1)\,KT_0B$$

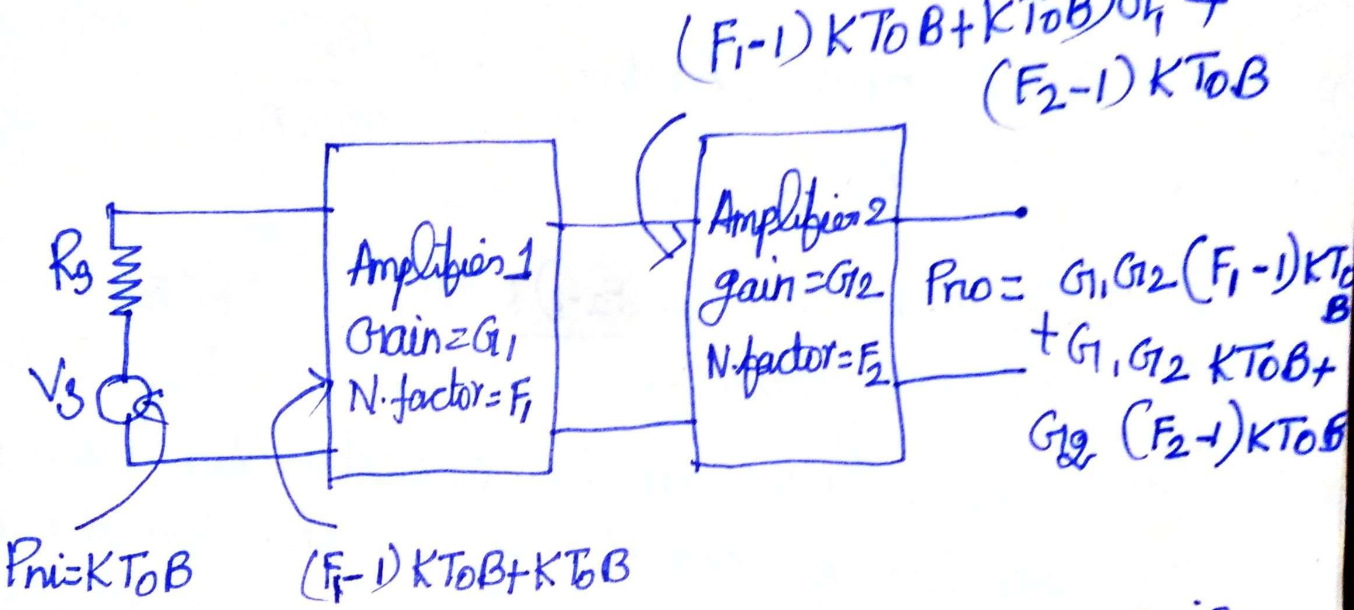
$$\boxed{T_{eq} = (F-1)\,T_0}$$

Noise Bandwidth

B is the Noise Bandwidth, which is the Bandwidth of the filter which an ideal rectangular amplitude response that passes the same power as the cascaded filters in the receiver.

Noise Factors of Amplifiers in cascaded form

In practice, the filters or amplifiers are not used isolated manner. They are used in cascaded manner.



The total noise power at i/p of first amplifier is given by

$$P_{ni}(\text{total}) = (F_1-1)KT_0B + KT_0B$$

The total noise power at o/p of amplifier 1 will be addition of 2 terms

$$\text{Noise i/p of amplifier } 2 = G_1(F_1-1)KT_0B + (F_2-1)KT_0B + G_1 KT_0B$$

The noise power at the output of second amplifier is

$$P_{no} = G_2 \times (\text{Noise i/p to 2 amplifier}$$

$P_{no} = G_{12} \times$ (Noise i/p to 2 amplifier)

$P_{no} = G_1 G_{12} F_1 K T_0 B + (F_2 - 1) K T_0 B G_{12}$

The overall gain of the cascade connection is given by

$$G = G_1 G_{12}$$

Overall noise factor $F = \frac{P_{no}}{G_1 G_{12} P_{ni}}$

$$P_{ni} = K T_0 B$$

$$F = \frac{G_1 G_{12} F_1 K T_0 B + G_{12} (F_2 - 1) K T_0 B}{G_1 G_{12} K T_0 B} = F_1 + \frac{(F_2 - 1)}{G_1}$$

The same logic can be extended for more number of amplifier is connected in cascade. Then the expression for overall noise factor F would be

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_{12}} + \frac{(F_4 - 1)}{G_1 G_{12} G_{13}} + \cdots\cdots$$

Equivalent Noise Temperature of Amplifiers in Cascade

The Friss formula derived for overall noise factor can be written in terms of overall noise temperature

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_{12}} + \cdots\cdots$$

Subtrating 1 from both sides, we get

$$(F-1) = (F_1-1) + \frac{(F_2-1)}{G_1} + \frac{F_3-1}{G_1 G_2} + \cdots$$

$$F-1 = \frac{T_{eq}}{T_0}$$

$$\frac{T_{eq}}{T_0} = \frac{T_{eq1}}{T_0} + \frac{T_{eq2}}{G_1 T_0} + \frac{T_{eq3}}{G_1 G_2 T_0} + \cdots\cdots$$

$$T_{eq} = T_{eq1} + \frac{T_{eq2}}{G_1} + \frac{T_{eq3}}{G_1 G_2}$$

where T_{eq1}, T_{eq2} are noise temperature of amplifiers 1,2 etc.

Representation of Narrow Band Noise

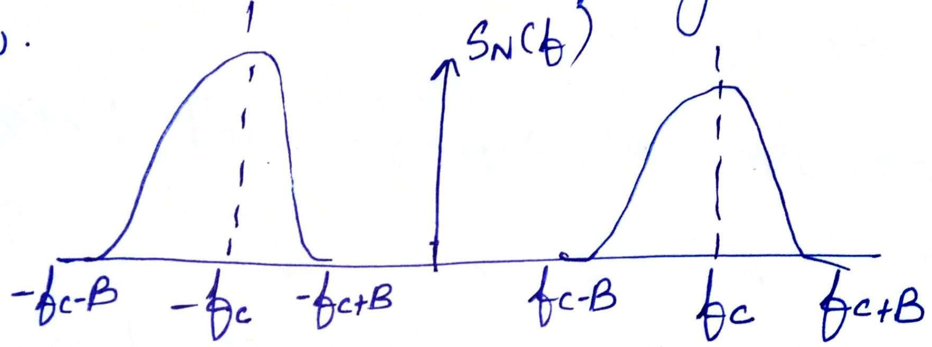
The front end of receiver of Communication System Consists of frequency Selective filters. The filters process the desired signal and Noise.

The filters are designed to have BW large enough to pass the signal without distortion but not to admit the noise through the receiver.

This filter is narrow Band i.e) B.w is small compared to mid frequency. the noise appearing at the O/P of this filter is called Narrow Band Noise.

Representation of Narrow Band Noise interms of ~~inphase and Quadrature Component~~ Envelope and phase

Consider a narrow band noise n(t) of B.w = 2B centered on frequency f_c as shown below.



Representation of n(t) in canonical form is

$$n(t) = n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t \rightarrow ①$$

Where

$n_I(t)$ - inphase Component of n(t)

$n_Q(t)$ - Quadrature Component of n(t)

~~Both~~ The probability distribution of r(t) and ψ(t) may be obtained from those of $n_I(t)$ and $n_Q(t)$.

Let n_I and N_Q are independent Gaussian variable of zero mean and variance σ^2, the joint probability density function is represented as

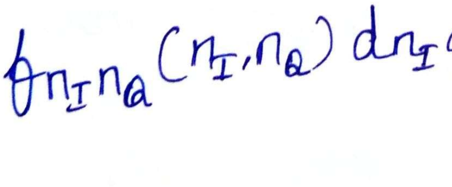
$$f_{N_I N_Q}(n_I, n_Q) = \frac{1}{2\pi\sigma^2} \exp\left(-\left(\frac{n_I^2+n_Q^2}{2\sigma^2}\right)\right) \rightarrow ②$$

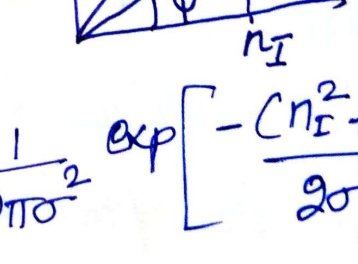
The probability of joint events lies between.

$$n_I \le N_I \le n_I + dn_i$$

$$n_Q \le N_Q \le n_Q + dn_Q$$

(i.e) The pair of random variable N_i and N_Q lie jointly inside the shared area of the fig. below is given by (differentiating)





$$f_{n_I n_Q}(n_I, n_Q)\, dn_I\, dn_Q = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{(n_I^2+n_Q^2)}{2\sigma^2}\right] dn_I dn_Q$$

↳ ③

from fig

$$n_I = r \cos\psi \rightarrow ④$$

$$n_Q = r \sin\psi \rightarrow ⑤$$

In the limiting sense we may equate the shaded areas in the above figures.

$$dn_Q dn_I = r\, dr\, d\psi \rightarrow ⑥$$

Sub 4, 5, 6 in ③

$$= \frac{1}{2\pi\sigma^2} \exp\left(\frac{-[r^2\cos^2\psi + r^2\sin^2\psi]}{2\sigma^2} \right) r\, dr\, d\psi$$

$$= \frac{1}{2\pi\sigma^2} \exp\left(\frac{-r^2}{2\sigma^2} \right) r\, dr\, d\psi$$

$$= \frac{r}{2\pi\sigma^2} \exp\left[\frac{-r^2}{2\sigma^2} \right] dr\, d\psi$$

Thus the joint pdf of R and ψ is

$$f_{R,\psi}(r,\psi) = \frac{r}{2\pi\sigma^2} \exp\left[\frac{-r^2}{2\sigma^2} \right] \rightarrow ⑦$$

The power density function is independent of ψ. Thus $f_{R,\psi}(r,\psi)$ can be expressed as the product of $f_R(r)$ and $f_\psi(\psi)$.

The pdf of ψ is

$$f_{\psi}(\psi) = \begin{cases} 1/2\pi & 0 \le \psi \le 2\pi \\ 0 & o.w \end{cases} \rightarrow ⑧$$

Pdf of R is

$$f_R(r) = \begin{cases} r/\sigma^2 \exp\left[\frac{-r^2}{2\sigma^2}\right], & r \ge 0 \\ 0 & o.w \end{cases} \rightarrow ⑨$$

where σ^2 is variance of $n(t)$. the pdf of ⑨ is said to be Rayleigh distribution

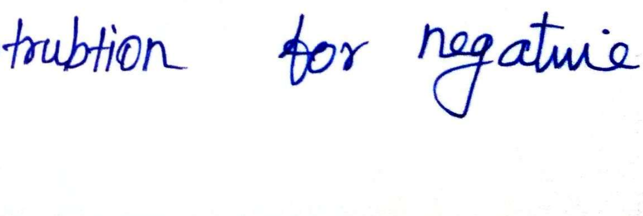
Let $\frac{r}{\sigma} = v$, $f_v(u) = \sigma f_R(r) \rightarrow ⑩$

Sub ⑩ in ⑨

$$f_v(u) = \sigma \begin{cases} v/\sigma \exp(-v^2/2), & v \ge 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$f_v(u) = \begin{cases} v \exp(-v^2/2) & v \ge 0 \\ 0 & o.w \end{cases} \rightarrow ⑪$$

Plotting equation ⑪

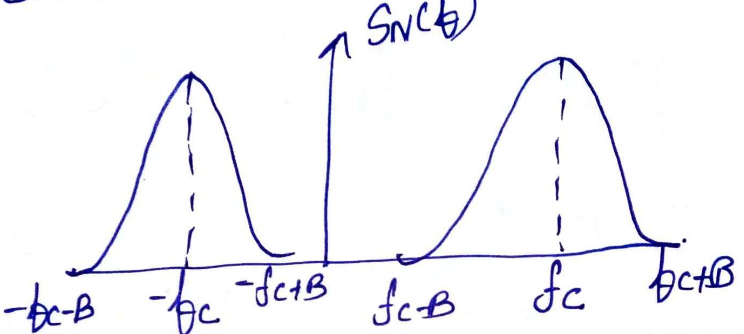


The peak value of the distributions occur at v=1. In Rayleigh distrubtion for negative values of v is zero.

Since r(t) can be assumed only positive.

Representation of narrowband noise interms of inphase and Quadrature Component

Consider a narrow band noise n(t) of Bandwidth = 2B centered on frequency fc as shown below.



Representation of n(t) in canonical form is

$$n(I) = n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$

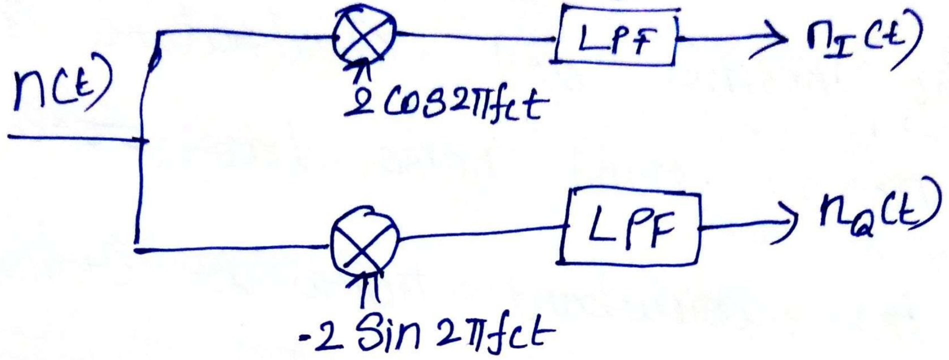
where $n_I(t) \rightarrow$ inphase Component of n(t)

$n_Q(t) \rightarrow$ Quadrature Component of n(t)

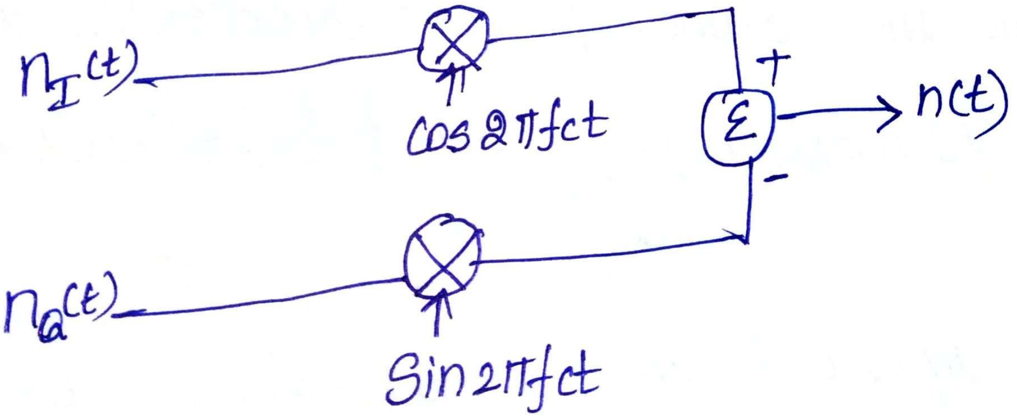
Both $n_I(t)$ and $n_Q(t)$ are low pass signals.

The inphase and Quadrature

Component can be extracted from the narrowband noise using the figure below.



It is assumed that the two LPF are ideal having a B.w equal to B[One-half of the B.w of n(t)]. The above schematic follows eqn ⓐ, we can use the same equation to generate n(t) given its inphase and Quadrature component.



The important properties of inphase and Quadrature Components are

1) The inphase and Quadrature component of narrow band noise has zero mean.

2) If the narrowband noise is Gaussian, then the inphase and Quadrature Components are jointly Gaussian.

3) If the narrowband noise n(t) is stationary then inphase and Quadrature Components are jointly stationary.

4) Both the inphase and Quadrature Component have the same power spectral density

$$S_{NI}(f) = S_{NQ}(f) = \begin{cases} S_N(f-f_c)+S_N(f+f_c), & -B \le f \le B \\ 0 & \text{o.w} \end{cases}$$

5) The inphase and Quadrature Component have the same variance as narrow band Noise.

6) The cross spectral density of the noise

Noise Performance Analysis in AM systems

(i) Channel SNR for AM signal.

Consider the AM Transmission that has both sidebands and a carrier. The Modulated signal is mathematically represented as

$$S(t) = A_c[1+K_a m(t)] \cos 2\pi f_c t \rightarrow ①$$

Where, $A_c \cos 2\pi f_c t \rightarrow$ Carrier signal.

$m(t)$ message signal, $K_a \rightarrow$ modulation index

Total power in modulated signal is given by

$$P_{total} = P_c\left[1+\frac{m_a^2}{2}\right]$$

$$= \frac{A_c^2}{2}\left[1+\frac{m_a^2}{2}\right]$$

Carrier power

$$P_c = \frac{A_c^2}{2}$$

$\therefore m_a = K_a$

$$P_{total} = \frac{A_c^2}{2}\left[1+\frac{K_a^2}{2}\right]$$

$\frac{K_a^2}{2}$ indicates the normalized power of message signal. If P is the average power of message signal then above equation becomes

$$P_{total} = \frac{A_c^2}{2}\left[1+K_a^2 P\right] \longrightarrow ②$$

If message B.W is B, the average noise power $= N_0 B$ → ③

$$(SNR)_C = \frac{\text{Modulated Signal Power}}{\text{Average Noise Power}}$$

$$= \frac{\frac{A_c^2}{2}(1+K_a^2 P)}{N_0 B}$$

$$(SNR)_C = \frac{A_c^2[1+K_a^2 P]}{2 N_0 B}$$

(ii) Output SNR for Envelope detector

The envelope detector consist of Modulated signal ~~g(t)~~ $s(t)$ plus noise $n(t)$

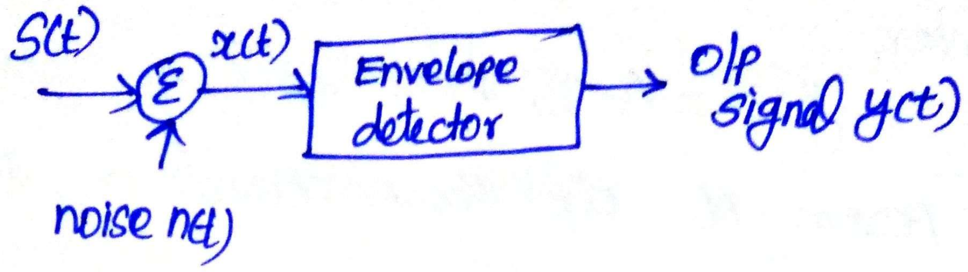
$$x(t) = s(t) + n(t)$$

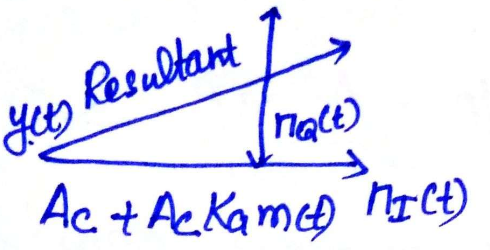
Representing $n(t)$ interms of inphase and Quadrature components

$$x(t) = s(t) + n_I \cos 2\pi f_c t - n_Q \sin 2\pi f_c t$$

$$= A_c[1 + K_a m(t)] \cos 2\pi f_c t + n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$

$$x(t) = [A_c + A_c K_a m(t) + n_I(t)] \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$





Phasor diagram of AM plus noise or phasor diagram of x(t)

The resultant is the envelope of x(t) (i.e) o/p of envelope detector is

$$y(t) = \sqrt{(A_c + A_c K_a m(t) + n_I(t))^2 + (n_Q(t))^2} \longrightarrow ⑤$$

When signal power is large compared to noise power. then $n_Q(t)$ & $n_I(t)$ will be very small compared to $A_c[1 + K_a m(t)]$

$$⑤ \Rightarrow y(t) = \sqrt{A_c (1 + K_a m(t))^2}$$

$$= A_c[1 + K_a m(t)]$$

$$y(t) = A_c + A_c K_a m(t)$$

The first term in above eqn is A_c. It is carrier amplitude and it can be removed with the help of blocking capacitor after envelope detector

$$y(t) = A_c K_a m(t) \rightarrow ⑥$$

The power of o/p above signal is average power at receiver o/p.

$$\text{Power at receiver o/p} = \frac{A_c^2 K_a^2 P}{2} \rightarrow ⑦$$

P→ Average power of message signal m(t)

$$\text{Noise power at receiver o/p} = N_0 B \rightarrow ⑧$$

$$(SNR)_o = \frac{\text{Power at receiver o/p}}{\text{Noise power at receiver o/p}}$$

$$= \frac{\frac{A_c^2 K_a^2 P}{2}}{N_0 B}$$

$$(SNR)_o = \frac{A_c^2 K_a^2 P}{2 N_0 B} \rightarrow ⑨$$

(iii) Figure of merit

$$F = \frac{(SNR)_o}{(SNR)_c}$$

$$= \frac{A_c^2 K_a^2 P}{2 N_0 B} \times \frac{2 N_0 B}{A_c^2 [1 + K_a^2 P]}$$

$$F = \frac{K_a^2 P}{1 + K_a^2 P}$$

For envelope detection, figure of merit is always less than unity.

Threshold effect

When the carrier to Noise ratio reduces below certain value, the message information is lost. The performance of envelope detector deteriorates rapidly and it has no proportion to carrier to noise ratio. This is called Threshold effect.

Every nonlinear receiver exhibits Threshold effect coherent receiver do not have threshold effect.

The detector output does not depends only on message signal m(t) but it is a function of noise also when the noise is higher compared to message signal. the noise dominates the performance of receiver.

Noise interms of envelope and phase Component is

$$n(t) = r(t) \cos (2\pi f_c t + \psi(t)$$

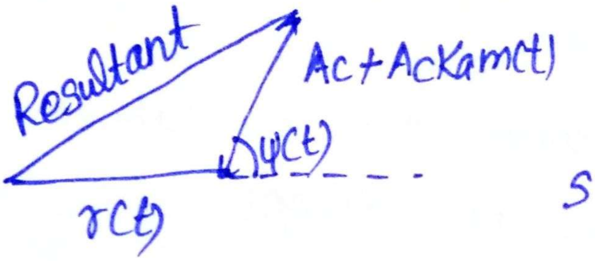
$r(t) \rightarrow$ Magnitude of noise, $\psi(t) \rightarrow$ phase of noise

In coherent detector

$$x(t) = S(t) + n(t)$$

$$= A_c\left[1 + K_a m(t)\right] \cos 2\pi f_c t + r(t) \cos\left[2\pi f_c t + \psi(t)\right]$$

$$x(t) = \left[A_c + A_c K_a m(t)\right] \cos 2\pi f_c t + r(t) \cos\left[2\pi f_c t + \psi(t)\right] \rightarrow ⑪$$



Phasor diagram of AM signal (The term r(t) is used as a reference)

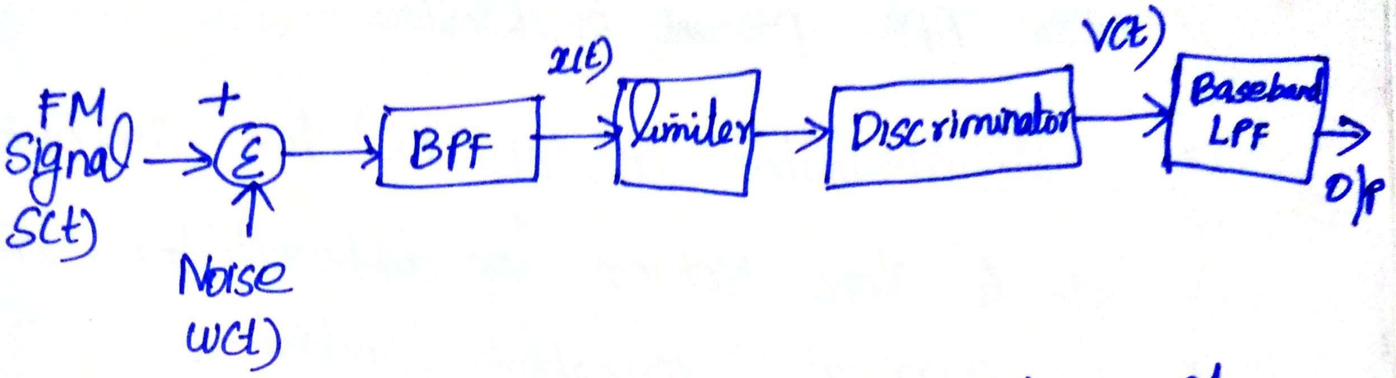
⑪ ⇒

$$x(t) = \left[A_c + A_c K_a m(t)\right] \cos 2\pi f_c t + r(t)\left[\cos 2\pi f_c t \cos \psi t - \sin 2\pi f_c t \sin \psi t\right]$$

$$x(t) = \left(A_c + A_c K_a m(t) + r(t) \cos \psi(t)\right] \cos 2\pi f_c t$$

$$r(t) \sin 2\pi f_c t \sin \psi(t)$$

Noise Performance Analysis in FM Systems



The Noise $w(t)$ is White Gaussian Noise of zero mean and power spectral density $N_0/2$

The FM signal $s(t)$ has a carrier frequency f_c and Bandwidth B_T.

B_T is small than f_c so we use narrow band noise $n(t)$

In FM the information is transmitted by variation of the instantaneous frequency of a sinusoidal carrier wave.

Therefore any variation of the carrier amplitude at the receiver input indicate the presence of noise.

The amplitude limiter following BPF is used to remove amplitude variations. The resulting rectangular wave is roundoff by another BPF present in limiter.

The discriminator consists of two components

1. A slope Network (or) differentiator with a purely imaginary transfer function.

2. An envelope detector that recovers the amplitude variation and thus reproduces the message signal.

The filter noise $n(t)$ is represented as

$$n(t) = n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$

Interms of envelope and phase.

$$n(t) = r(t) \cos (2\pi f_c t + \psi(t))$$

where $r(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$

$$\psi(t) = \tan^{-1} \frac{n_Q(t)}{n_I(t)}$$

The incoming FM signal is

$$S(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

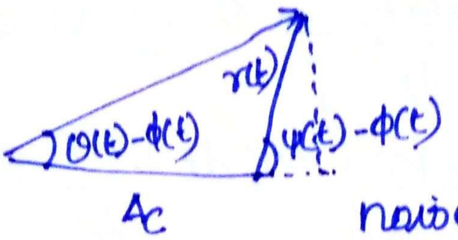
$$s(t) = A_c \cos\left[2\pi f_c t + \int_0^t 2\pi K_f m(t) dt\right]$$

$$\therefore \phi(t) = 2\pi K_f \int_0^t m(t) dt$$

The Output of BPF is

$$x(t) = s(t) + n(t)$$

$$= A_c \cos(2\pi f_c t + \phi(t)) + r(t) \cos[2\pi f_c t + \psi(t)] \rightarrow ⑤$$



Phasor diagram for FM wave plus narrowband noise for the case of high carrier to Noise ratio.

$$\cos(\psi(t) - \phi(t)) = \frac{\text{adjacent side}}{r(t)} \Rightarrow \text{adj side} = r(t)\cos[\psi(t) - \phi(t)]$$

$$\sin[\psi(t) - \phi(t)] = \frac{\text{opposite side}}{r(t)} \Rightarrow \text{opposite side} = r(t)\sin[\psi(t) - \phi(t)]$$

$$\tan[\theta(t) - \phi(t)] = \frac{\text{opposite side}}{A_c + \text{adj. side}}$$

$$= \frac{r(t)\sin(\psi(t) - \phi(t))}{A_c + r(t)\cos(\psi(t) - \phi(t))}$$

$$\theta(t) = \phi(t) + \tan^{-1}\left\{\frac{r(t)\sin(\psi(t) - \phi(t))}{A_c + r(t)\cos[\psi(t) - \phi(t)]}\right\} \rightarrow ⑥$$

Let R be a random variable observed for envelope process and it is observed that R<Ac for more times.

$$\therefore \theta(t) = \phi(t) + \frac{r(t)}{A_c}\sin(\psi(t) - \phi(t)) \rightarrow ⑦$$

Sub the value of $\phi(t)$.

$$\theta(t) = 2\pi K_f \int_0^t m(t)\,dt + \frac{r(t)}{A_c}\sin(\psi(t)-\phi(t)) \rightarrow \textcircled{8}$$

The output of discriminator is

$$v(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} \rightarrow \textcircled{9}$$

$$= \frac{1}{2\pi}\frac{d}{dt}\left(2\pi K_f \int_0^t m(t)\,dt + \frac{r(t)}{A_c}\sin(\psi(t)-\phi(t))\right)$$

$$= \frac{1}{2\pi} \times 2\pi K_f \frac{d}{dt}\int_0^t m(t)\,dt + \frac{1}{2\pi A_c}\frac{d}{dt}(r(t)\sin(\psi(t)-\phi(t)))$$

$$= K_f m(t) + \frac{1}{2\pi A_c}\frac{d}{dt}(r(t)\sin(\psi(t)-\phi(t)))$$

$$v(t) = K_f m(t) + n_d(t) \rightarrow \textcircled{10}$$

$$\therefore n_d(t) = \frac{1}{2\pi A_c}\frac{d}{dt}(r(t)\sin(\psi(t)-\phi(t)))$$

The noise at discriminator o/p is independent of message component

$$n_d(t) = \frac{1}{2\pi A_c}\frac{d}{dt}(r(t)\sin\psi(t)) \rightarrow \textcircled{11}$$

Noise interms of inphase and Quadrature Component

$$n(t) = n_I(t)\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t \rightarrow \textcircled{12}$$

Noise interms of envelope and phase component

$$n(t) = r(t)\cos[2\pi f_c t + \psi(t)]$$

$$= r(t)[\cos 2\pi f_c t \cos\psi(t) - \sin 2\pi f_c t \sin\psi(t)] \rightarrow \textcircled{13}$$

Comparing 12& 13

$$n_Q(t) = r(t)\,\sin\psi(t) \rightarrow ⑭$$

Sub ⑭ in ⑪

$$n_d(t) = \frac{1}{2\pi A_c}\frac{d}{dt}\,n_Q(t) \rightarrow ⑮$$

from eqn ⑮

Average signal power $= k_f^2 P \rightarrow$ ⑯

Average Noise Power is given by

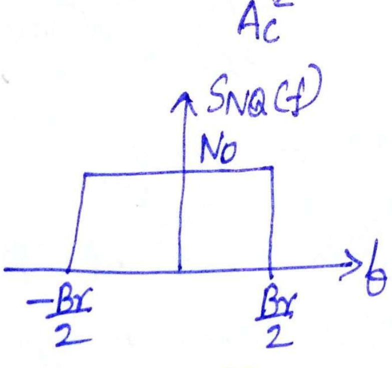
⑮ $\Rightarrow n_d(t) = \frac{1}{2\pi A_c} \times j2\pi f$ [∴ differentiation of any fn with respect to time is multiplication of Fourier transform by $j2\pi f$]

$$n_d(t) = \frac{jf}{A_c} \rightarrow ⑰$$

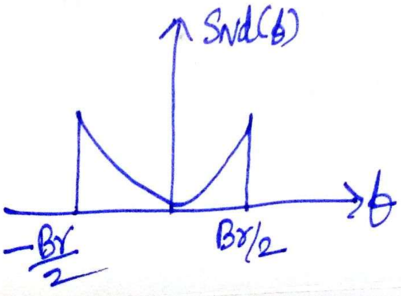
Therefore we obtain $n_d(t)$ by passing $n_Q(t)$ through a linear filter with a transfer function jf/f_c.

Power Spectral density is

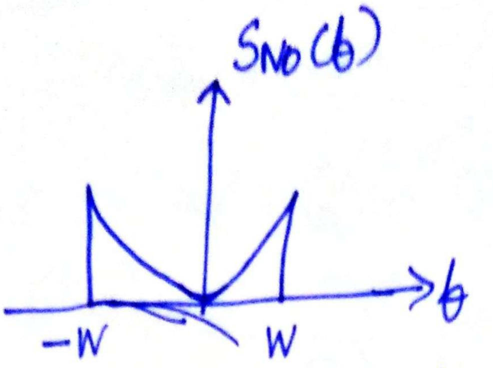
$$S_{Nd}(f) = \frac{f^2}{A_c^2}\,S_{NQ}(f)$$



(a) Power Spectral density of quadrature component $n_Q(t)$ of narrowband noise $n(t)$



(b) Power Spectral density of $n_d(t)$ of discriminator



(c) Power Spectral density of noise $n_o(t)$ at receiver o/p

Power Spectral density of $n_d(t)$ is

$$S_{Nd}(f) = \begin{cases} \frac{N_o f^2}{A_c^2}, & |f| \le B_T/2 \\ 0 & \text{o.w} \end{cases}$$

The Output of LPF having B.W $\omega < B_T/2$ rejects the out of band components of $n_d(t)$

$$S_{NO}(f) = \begin{cases} \frac{N_o f^2}{A_c^2}, & |f| \le W \\ 0 & \text{o.w} \end{cases}$$

$$\text{Average o/p Noise Power} = \int_{-\omega}^{W} \frac{N_o f^2}{A_c^2}\, df$$

$$= \frac{N_o}{A_c^2}\left[\frac{f^3}{3}\right]_{-W}^{W}$$

$$= \frac{N_o}{3A_c^2}\left[W^3 + W^3\right]$$

$$= \frac{2 N_o W^3}{3A_c^2}$$

$$(SNR)_o = \frac{\text{Average Signal Power at O/P}}{\text{Average Noise Power at O/P}}$$

$$= \frac{K_f^2 P \times 3A_c^2}{2N_o\omega^3}$$

$$(SNR)_0 = \frac{3A_c^2 K_f^2 P}{2N_0 W^3}$$

SNR at the channel

$$(SNR)_C = \frac{\text{Average Power of Message Signal at receiver O/P}}{\text{Average Power of Noise in message BW at receiver I/P}}$$

$$= \frac{A_c^2/2}{WN_0}$$

$$(SNR)_c = \frac{A_c^2}{2WN_0}.$$

Figure of Merit $\nu = \frac{(SNR)_0}{(SNR)_C}$

$$\nu = \frac{3A_c^2 K_f^2 P}{2N_0 W^3} \times \frac{2WN_0}{A_c}$$

$$\nu = \frac{3K_f^2 P}{W^2}$$

The deviation ratio $D = \frac{\Delta f}{W} = \frac{\text{frequency deviation}}{\text{Message Bandwidth.}}$

$$D = \frac{K_f P^{1/2}}{W}$$

Capture effect

The FM system minimize the effect of noise interference. This can be effective when interference is weak compared to FM signal.

But if the interference is stronger than FM signal, the FM receiver looks to interference. This

Suppresses FM signal.

When Noise interference as well as FM Signal are of equal strength. then the FM receiver locking fluctuates between them. This phenomenon is called Capture effect.

FM Threshold effect

It is the minimum carrier to noise ratio yielding an FM improvement which is not significantly deteriorated from the value predicted by the usual SNR formula assuming small noise

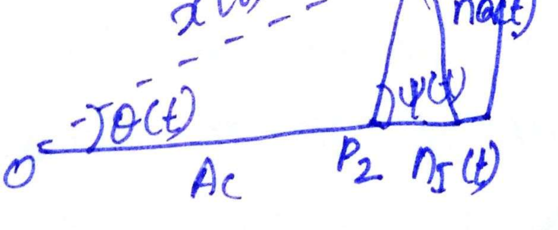
Consider carrier is unmodulated. The signal at O/P of discriminator is represented as

$$x(t) = s(t) + n(t) \rightarrow ①$$

$$s(t) = A_c \cos 2\pi f_c t \text{ with no modulation}$$

$$x(t) = A_c \cos 2\pi f_c t + n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$

$$x(t) = [A_c + n_I(t)] \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t \rightarrow ②$$



The amplitude of $n_I(t)$ and $n_Q(t)$ changes with time in a random manner. The point P_1 wanders around the point P_3.

When carrier to noise ratio is large $n_I(t)$ & $n_Q(t)$ are usually smaller than A_c and P_1, spend most of its time P_2.

From fig. $\tan\theta = \frac{n_Q(t)}{A_c + n_I(t)}$

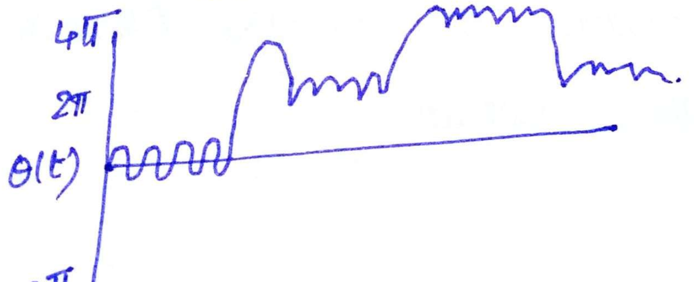
$$\theta = \tan^{-1}\left(\frac{n_Q(t)}{A_c}\right)$$

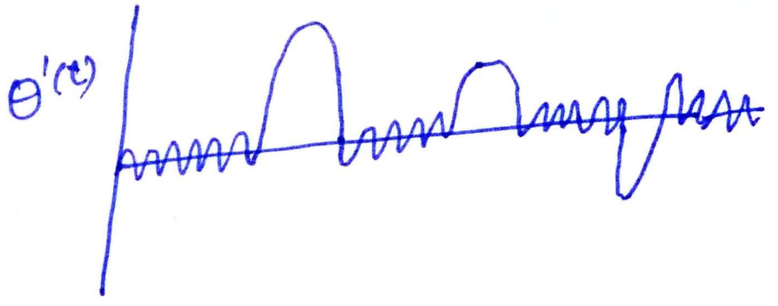
$\theta = \frac{n_Q(t)}{A_c} \Rightarrow \theta$ lies within a multiple of 2π

When carrier to Noise ratio is less P_1 sweeps around the origin and $\theta(t)$ increases (or) decreases by 2π radian.

Discriminator o/p is

$$\frac{\theta'(t)}{2\pi} = \frac{1}{2\pi}\frac{d\theta}{dt}.$$





The height of impulse depends on the wandering point P_1.

When this signal is passed through LPF: the impulses are excited h. 'click sound. The clicks are produced only when $\theta(t)$ changes by $\pm 2\pi$ radians.

positive going click

Condition for occurance of click.

(i) $r(t) > A_c$

(ii) $\psi(t) < \pi < \psi(t) + d\psi(t)$, $\frac{d\psi(t)}{dt} > 0$

Condition for negative click

(i) $r(t) > A_c$

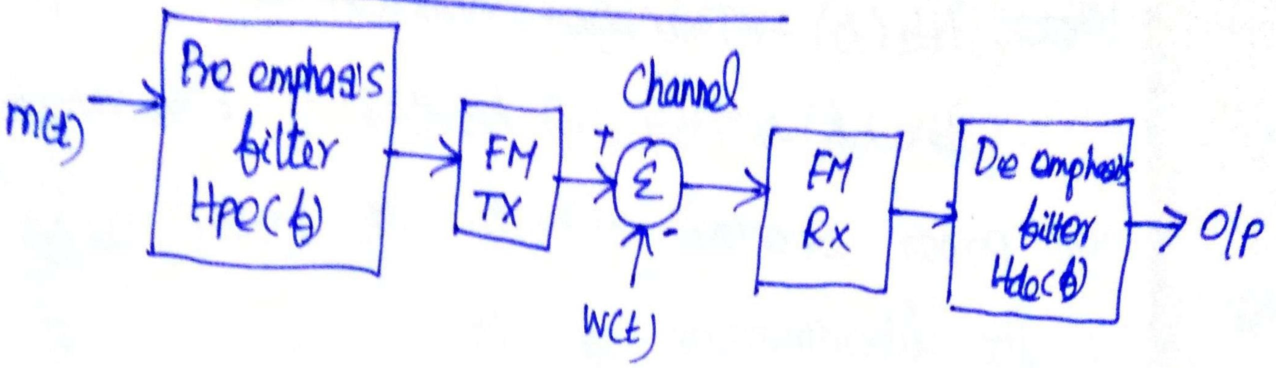
(ii) $\psi(t) < -\pi < \psi(t) + d\psi(t)$, $\frac{d\psi(t)}{dt} < 0$

The carrier to Noise ratio is defined as

$$\rho = \frac{A_c^2}{2B_T N_0}$$

As ρ decreases the average number of clicks per unit time decreases.

Pre-emphasis and De-emphasis



The high frequency components are artificially emphasized by pre-emphasis filter before Modulation. This equalizes the low frequency and high frequency portions of PSD and complete band is occupied.

The FM signal is then transmitted. Noise adds to this signal before it reaches receiver.

At receiver de-emphasis is performed on high frequency components. This restores the power distributions of original signal.

Because of De-emphasis at receiver, high frequency components of noise are also reduced. This improves SNR.

In order to obtain the original signal back, the transfer function of pre emphasis and de emphasis filters must be inverse of each other.

$$H_{de}(f) = \frac{1}{H_{pe}(f)}, \quad -W \leq f \leq W$$

Where, $H_{de}(f) \rightarrow$ Transfer function of de-emphasis filter

$H_{pe}(f) \rightarrow$ Transfer function of pre emphasis filter

The power spectral density of noise $n_d(t)$ at the discriminator o/p is

$$S_{Nd}(f) = \begin{cases} \frac{N_0 f^2}{A_c^2}, & |f| \leq B_T/2 \\ 0 & O.\omega \end{cases}$$

Modified power spectral density of Noise at the de emphasis filter o/p is

$$|H_{de}(f)|^2 S_{Nd}(f)$$

$$= \frac{N_0}{A_c^2} \int_{-W}^{W} f^2 |H_{de}(f)|^2 df$$

$$I = \frac{\text{Avg o/p Noise Power without Pre emphasis \& de emphasis}}{\text{Avg o/p Noise power with pre emphasis \& de emphasis}}$$

$$= \frac{\frac{2N_0W^3}{3A_c^2}}{\frac{N_0}{A_c^2} \int_{-W}^{W} f^2 |H_{de}(f)|^2 df}$$

$$= \frac{2W^3}{3\int_{-W}^{W} |H_{de}(f)|^2 df}.$$

This improvement factor assumed the use of a high Carrier to Noise ratio.

Unit-V

Sampling & Quantization

Low Pass Sampling - Aliasing - Signal Reconstruction - Quantization - Uniform and Non uniform Quantization - Quantization Noise - Logarithmic Companding - PAM, PPM, PWM, PCM, TDM, FDM.

Why Digital Communication

Due to advancements in VLSI technology, it is possible to manufacture high speed embedded circuits. Such circuits are used in digital Communication.

High speed computers are powerful software design tools are available. They make digital Communication easier.

The Compatibility of digital Communication Systems with internet has opened new area of applications.

Advantages of Digital Communication

- * Simpler and cheaper due to high speed Computers and Ic Technology.
- * Regeneration of signal at receiver is easy.
- * Security since data encryption can be used.
- * Error detection & correction
- * Multiplexing can be used.

An analog signal can be converted into digital form by three basic operations (1) Sampling (2) Quantizing and (3) encoding

Sampling :- Only sample values of analog signal at uniformly spaced time intervals retained.

Quantizing :- Each sample value is approximated by the nearest level in a finite set of discrete levels.

Encoding :- Selected level is converted to a Codeword. Codewords are four binary digits (bits). Last bit represents the sign + or -.

Low Pass Sampling

Sampling is defined as the Process of Converting a Continuous time signal into a discrete time Signal by measuring the signal at periodic instants of time.

Consider an analog Signal $g(t)$ that is Continuous in both time and amplitude. $g(t)$ has infinite duration but finite energy.

Let the sample values of the signal $g(t)$ at times $t=0, \pm T_S, \pm 2T_S \ldots$ be denoted by the Series $\{g(nT_S), n=0, \pm 1, \pm 2 \ldots\}$.

Where $T_S \rightarrow$ sampling period and $f_S = 1/T_S$ Sampling rate.

The final discrete time Signal which is the result of Sampling is given by $g_S(t)$.

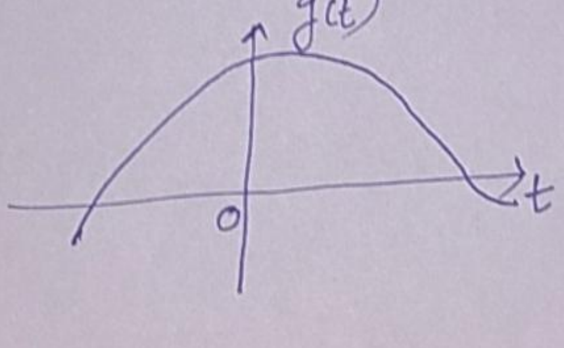


Fig:- Analog Signal.

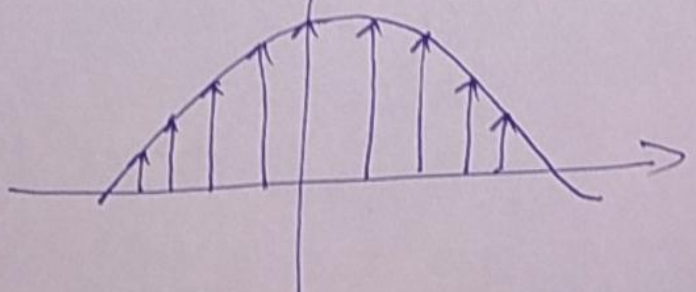


Fig: Discrete time Signal

$g_\delta(t)$ can be defined as the product of $g(t)$ and Dirac delta function $\delta_{T_s}(t)$

Thus we get

$$g_\delta(t) = g(t) \cdot \delta_{T_s}(t) \rightarrow ①$$

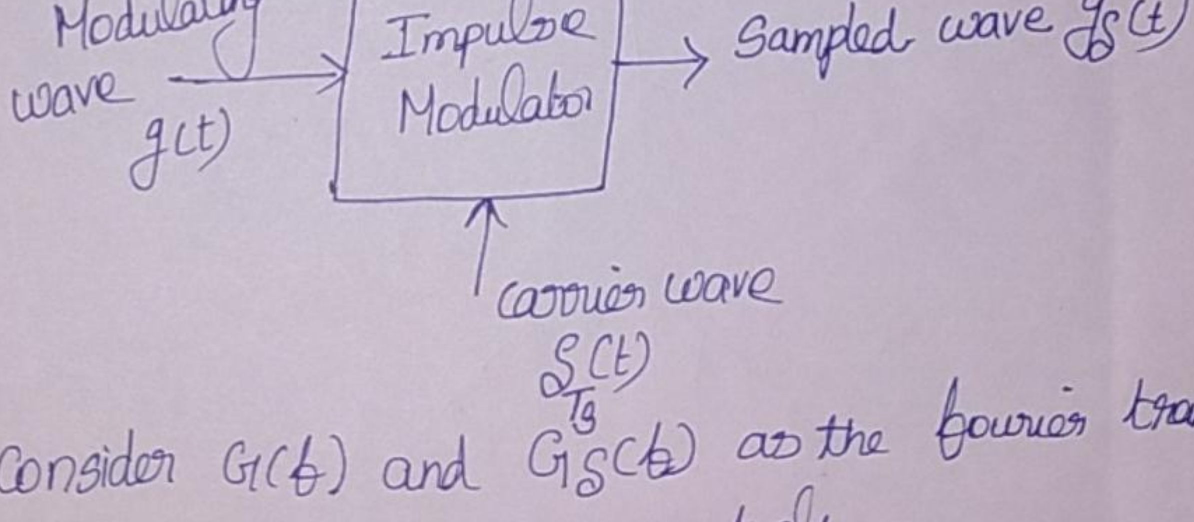
The delta function $\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_s) \rightarrow ②$

Substitute eqn ② in ①

$$g_\delta(t) = g(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$$

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(t)\,\delta(t-nT_s) \longrightarrow ③$$

equation ③ shows that $g_\delta(t)$ can be obtained as the output of an impulse modulator which takes $g(t)$ as the modulating wave and $\delta_{T_s}(t)$ as the carrier wave.



Consider $G(f)$ and $G_\delta(f)$ as the fourier transfor of $g(t)$ and $g_\delta(t)$ respectively

And the fourier transform $\delta_{TS}(t)$ is given by

$$F\left[\delta_{TS}(t)\right] = fs \sum_{m=-\infty}^{\infty} \delta(f-mfs)$$

Thus eqn ③ can be transformed to frequency domain as

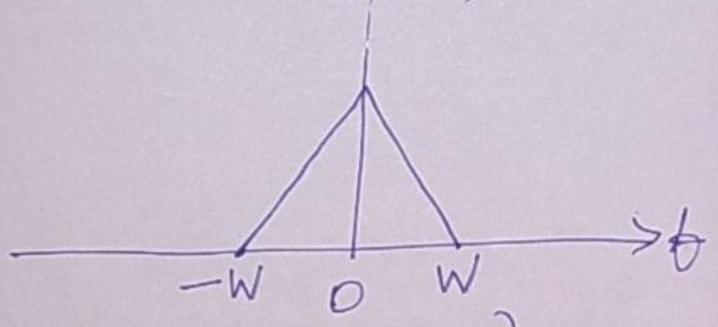
$$G_\delta(f) = G(f) * \left[fs \sum_{m=-\infty}^{\infty} \delta(f-mfs)\right]$$

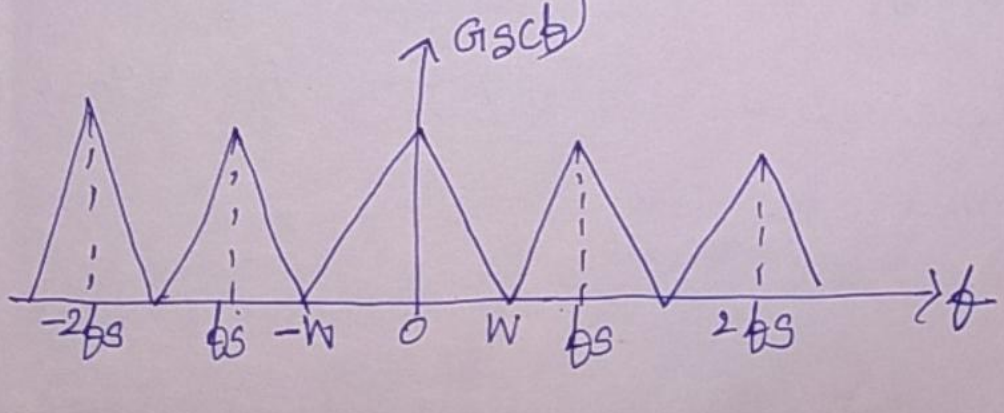
$$G_\delta(f) = fs \sum_{m=-\infty}^{\infty} G(f) * \delta(f-mfs)$$

$$G_\delta(f) = fs \sum_{m=-\infty}^{\infty} G(f-mfs) \quad \text{∴ By properties of a delta function}$$

↳④

Equation ④ of $G_\delta(f)$ represents a spectrum that is periodic in the frequency f with period fs. Thus $G_\delta(f)$ represents a periodic extension of the original spectrum $G(f)$.





Another expression for the fourier transform of $G_\delta(f)$ in terms of $g(nT_s)$ is given by

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s) \exp(-j2\pi n f T_s) \longrightarrow ⑤$$

Substitute $T_s = 1/2W$ in eqn ⑤

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(\frac{-j2\pi n f}{2W}\right)$$

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(\frac{-j\pi n f}{W}\right) \rightarrow ⑥$$

Substitute $f_s = 2W$ in eqn ④

$$G_\delta(f) = 2W \sum_{m=-\infty}^{\infty} G(f - mf_s)$$

$$G_\delta(f) = 2W\, G(f) \qquad \therefore -W < f < W$$

$$G(f) = \frac{1}{2W} G_\delta(f) \longrightarrow ⑦$$

Substitute eqn ⑥ in eqn ⑦

$$G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(\frac{-j\pi n f}{W}\right)$$

$\rightarrow$ ⑧

Therefore, if the sample values $g(n/2W)$ of the signal $g(t)$ are specified for all time, then the fourier transform $G(f)$ of the signal can be determined by using eqn ⑧

Signal Reconstruction

Consider reconstructing the signal g(t) from the sequence of sample values $\{g(n/2W)\}$.

Substitute eqn ⑧ in the formula for Inverse fourier transform,

$$g(t) = \int_{-\infty}^{\infty} G(f) \exp(j2\pi ft)\, df$$

$$= \int_{-W}^{W} \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(\frac{-j\pi nf}{W}\right) \exp(j2\pi ft)\, df$$

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{1}{2W} \int_{-W}^{W} \exp\left[j2\pi f\left(t-\frac{n}{2W}\right)\right] df.$$

By solving the above integration we get

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{\sin(2\pi Wt - n\pi)}{2\pi Wt - n\pi} \quad \to ⑨$$

The above equation of g(t) can be simplified by using sinc function which is defined as

$$\text{sinc}\, x = \frac{\sin(\pi x)}{\pi x} \qquad x \to \text{independent variable}$$

The sinc function has an important interpolatory property which is as follows

$$\text{sinc}\, x = \begin{cases} 1 & ; x = 0 \\ 0 & ; x = \pm 1, \pm 2, \ldots \end{cases}$$

Thus using Sinc function eqn ⑨ Can be written as

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \text{sinc}\,(2Wt-n) \quad \hookrightarrow ⑩$$

Thus eqn ⑩ provides an interpolation formula for reconstructing the original signal g(t) from the sequence of samples values $g(\frac{n}{2W})$.

Each sample $g(\frac{n}{2W})$ is multiplied by a delayed version of sinc function which is interploation function, and all the resulting waveforms are added to obtain g(t). This can be achieved by passing the samples ~~of~~ through an ideal low pass filter of Bandwidth.

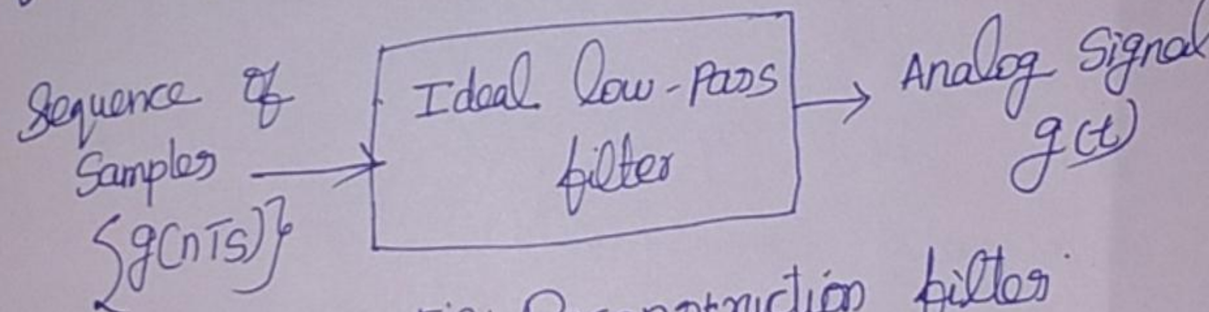


Fig:- Reconstruction filter

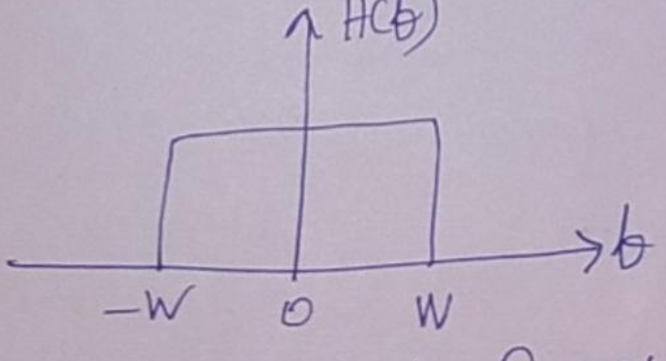
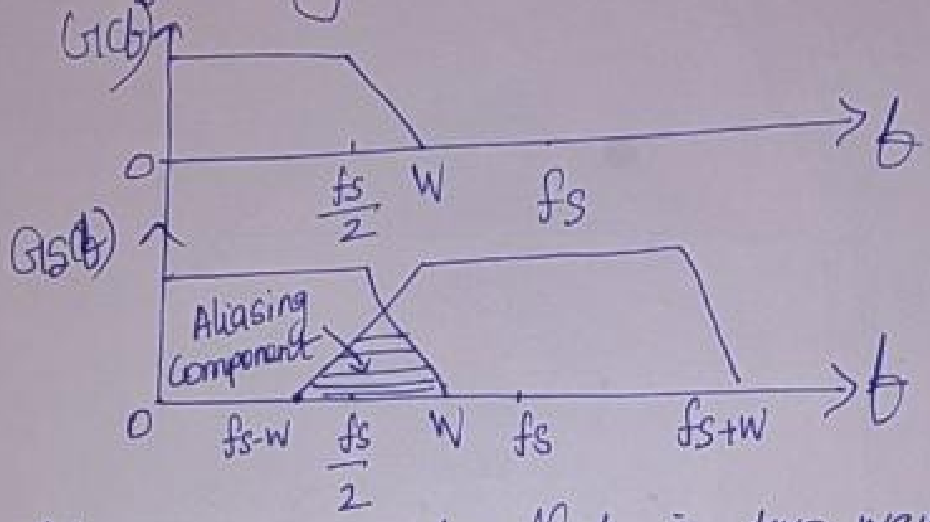


Fig:- Response of Reconstruction filter

Aliasing

In sampling a continuous analog signal $g(t)$ if $f_s < 2W$, then the sampling is referred as under sampling. As a result of undersampling the spectral components of $G_s(f)$ overlaps with the neighbouring components. This is called aliasing. (or) fold over.



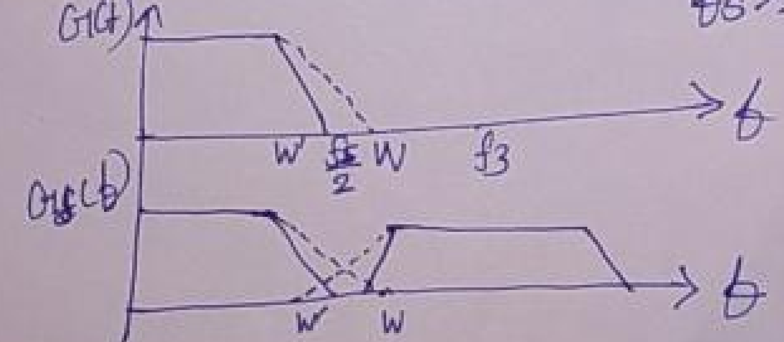
Aliasing can be handled in two ways

- *pre filtering } Anti aliasing filters
- *post filtering }

Pre filtering

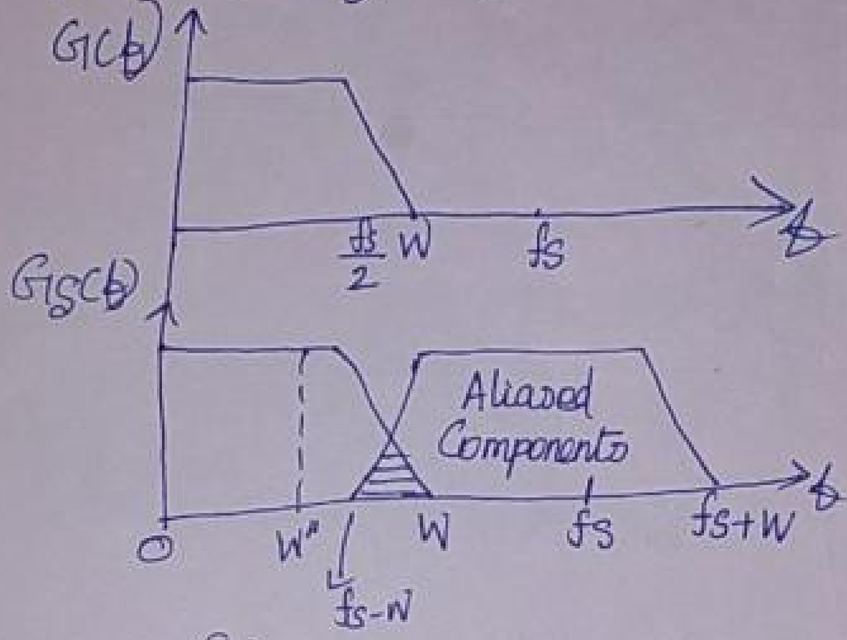
The analog signal is pre filtered so that the new minimum frequency W' is reduced to $\frac{f_s}{2}$ or less. Thus there are no aliased components since

$$f_s > 2W' \Rightarrow \frac{f_s}{2} > W' \Rightarrow W' = \frac{f_s}{2}$$



Post filtering

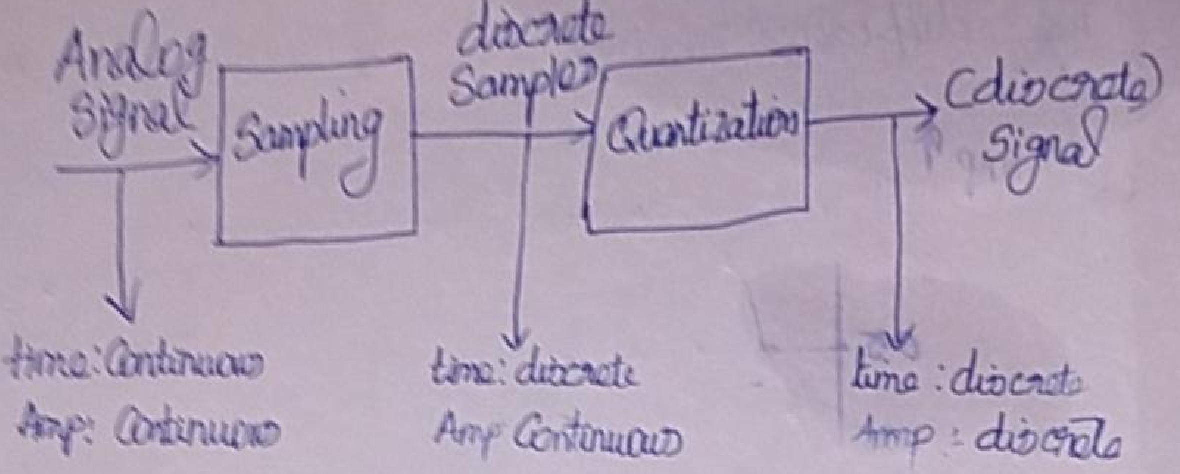
The aliased components can be removed by post filtering after sampling. The filter cut off frequency w'' removes the aliased components, where $w'' < (f_s - w)$. Both pre filtering and post filtering may result in information loss.



Over Sampling

When the analog signal is sampled at a rate, $f_s > 2W$ then the sampling is referred as over sampling

Quantization



Defn: The Conversion of an analog (continuous) Sample of the signal into a digital form is Called the quantizing process

Quantizing Process has a two-fold effect

1) peak to peak range of input sample value is subdivided into finite set of division levels or decision thresholds.

2) The output is assigned a discrete value from finite set of representation levels or reconstruction values.

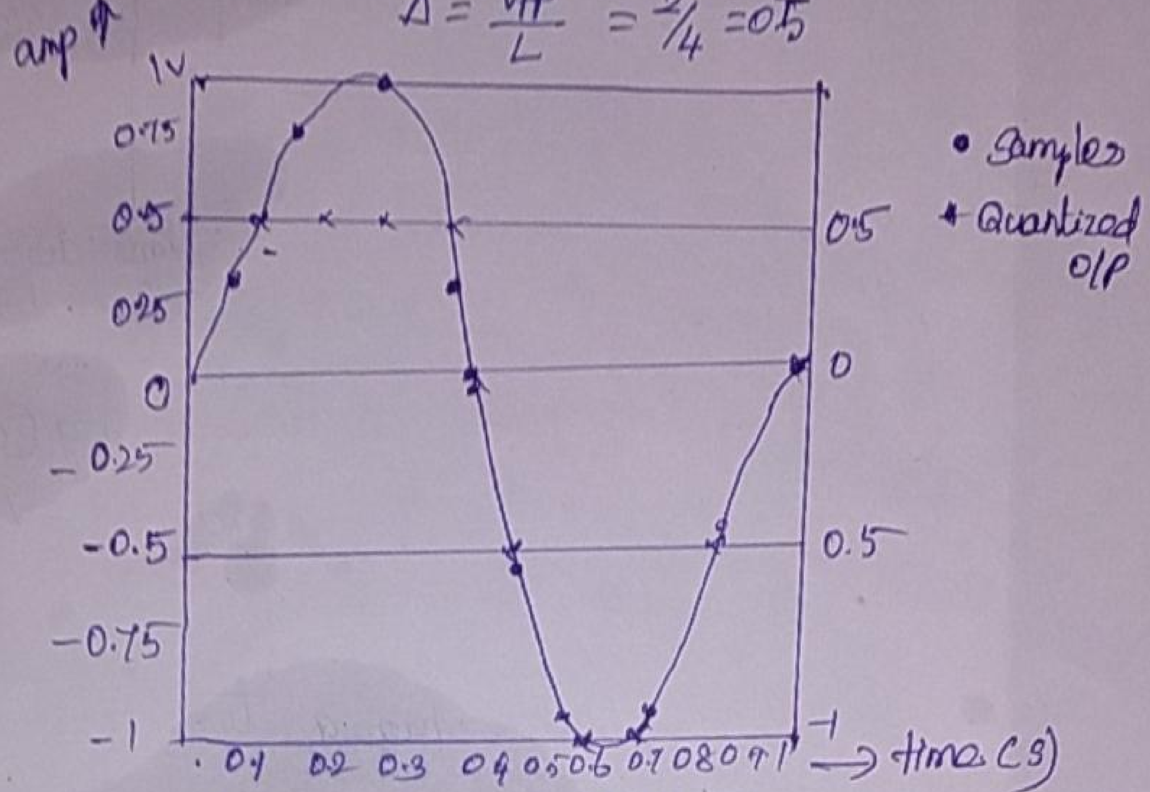
Quantization Example

Consider a sine wave of 1 Hz and 2 Vpp. It is sampled at a rate of 10 Samples/sec. It is Quantized with two bits [n=2].

So, the Quantization level, $L = 2^n \Rightarrow L = 4$.

The difference between each level (i.e) step size is

$$\Delta = \frac{V_{PP}}{L} = 2/4 = 0.5$$



Thus the four decision levels from -1V are -1, -0.5, 0 and +0.5. They are spread 0.5 ⇒ Δ apart. Now the samples are rounded off to the nearest decision level and the representation levels are marked. Since L=4, the representation levels are -1, -0.5, 0 & 0.5. All the discrete samples of the signal takes any one of the representation levels after round off.

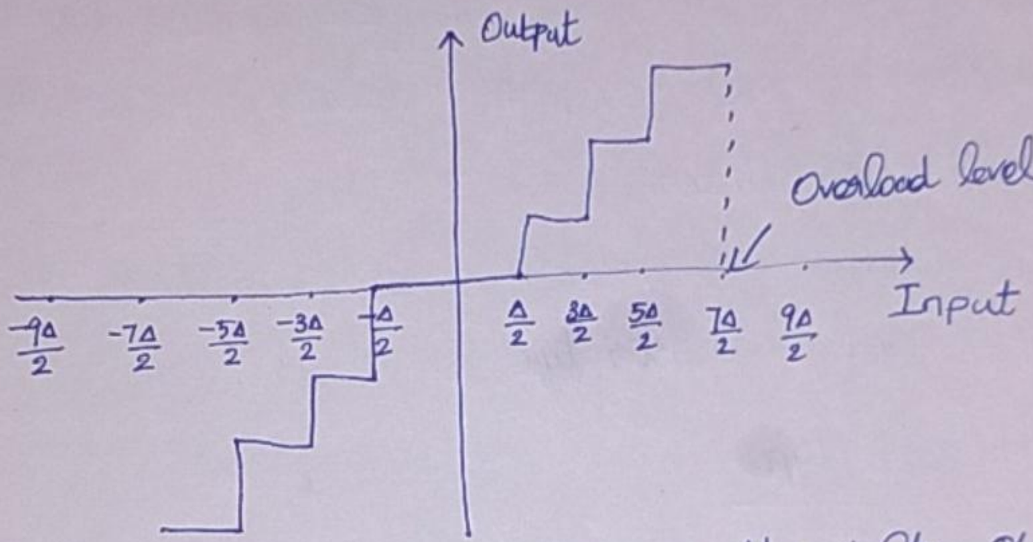
Thus the discrete samples of sampling are converted into digital form by Quantization.

Step size

The separation between the decision thresholds and the separation between the representation levels of the Quantizer have a common value called step size.

Transfer characteristics

The transfer characteristics of a Quantizer is staircase - like in appearance as shown in fig below.



In fig. above, the decision thresholds of the quantizer are located at $\pm\Delta/2$, $\pm\frac{3\Delta}{2}$, $\pm\frac{5\Delta}{2}$, ... and the representation levels are located at $0, \pm\Delta \pm 2\Delta$ ..... where Δ is the step size.

Quantization error

It is the difference between the Output and input values of the Quantizer.

A Quantizer is memoryless because the Quantizer Output is determined only by the value of a corresponding input Sample.

Types of Quantization

- Uniform Quantization
- Non uniform Quantization

Uniform Quantization

In this type of Quantization the step size is equal all over the transfer characteristics of the Quantizer.

Uniform Quantization is classified into midtread type and midriser type

Midtread type

In the transfer characteristics of a Quantizer, in the decision thresholds are located at $\pm \Delta/2, \pm \frac{3\Delta}{2}$ and representation levels are located at $0, \pm\Delta, \pm 2\Delta$ and if the origin lies in the middle of a tread of the Staircase, then the Quantizer is midtread.

Midriser type

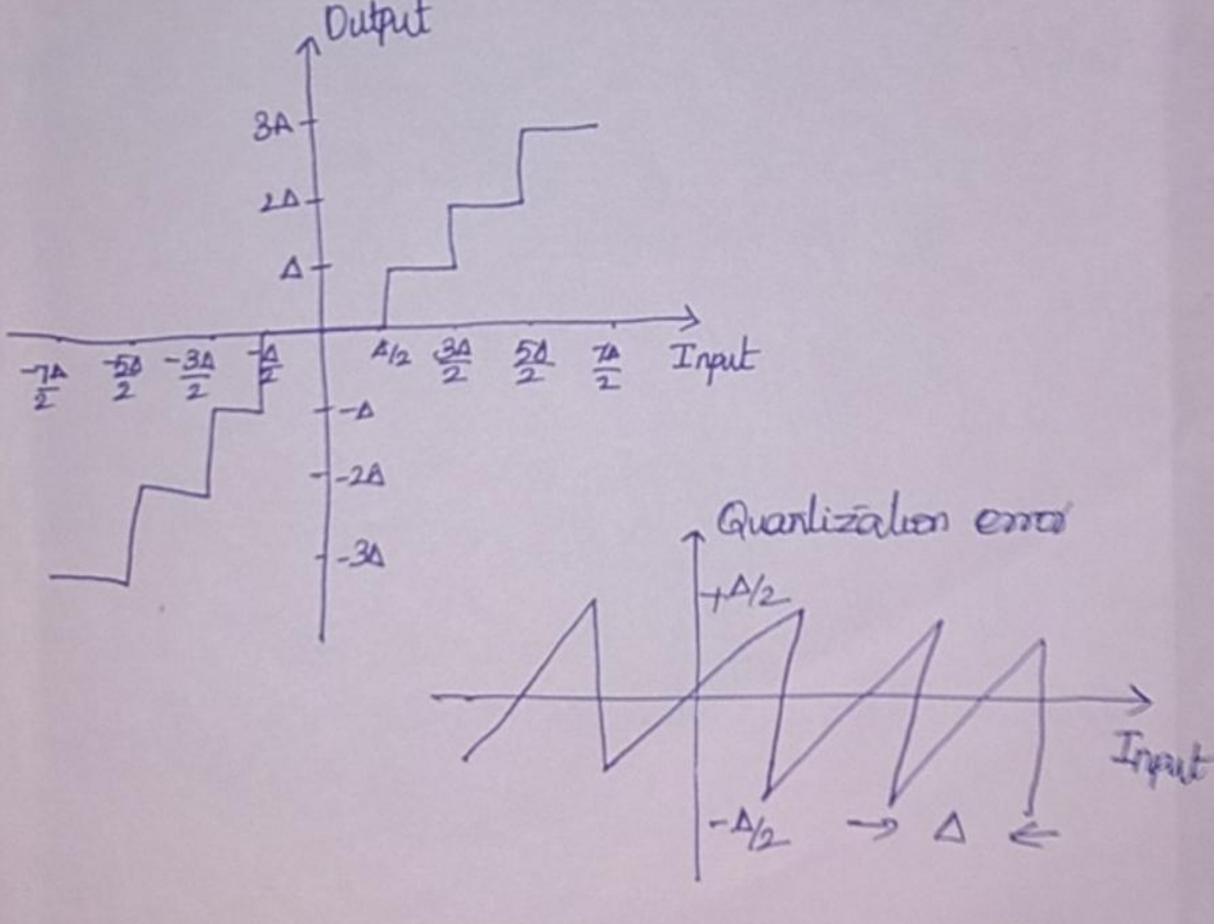
In the transfer characteristics of a Quantizer, if the decision thresholds are located at $0, \pm\Delta, \pm 2\Delta . . . .$ and representation levels are located at $\pm\Delta/2, \pm 3\Delta/2, \pm\frac{5\Delta}{2}$ and if the origin lies in the middle of the riser, then the Quantizer is of midriser type.

Overload level

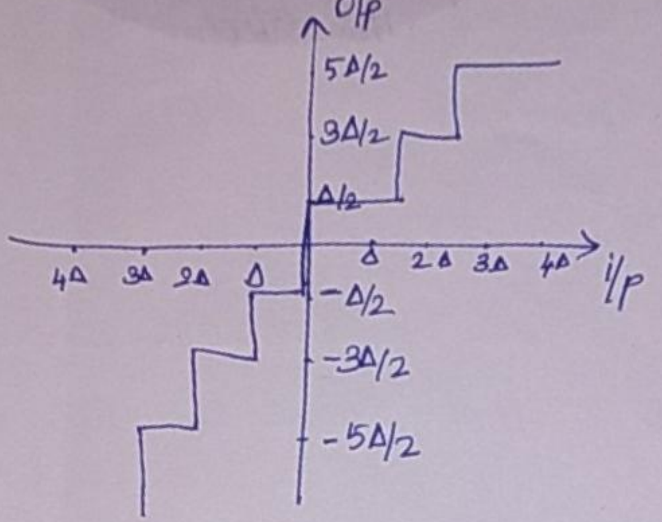
The absolute value of overload level is one half of the peak to peak range of input sample values.

Transfer characteristics of Uniform Quantization

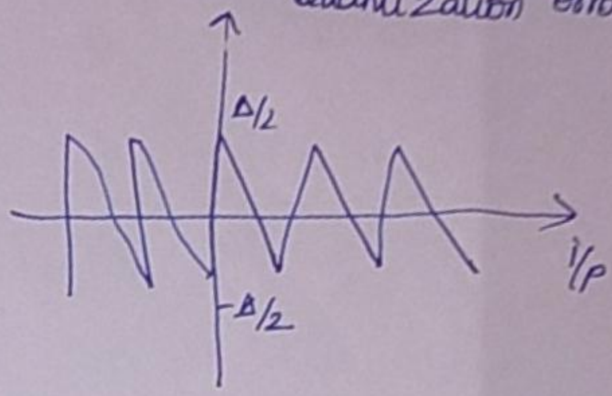
Midtread type



Midriser type



Quantization error



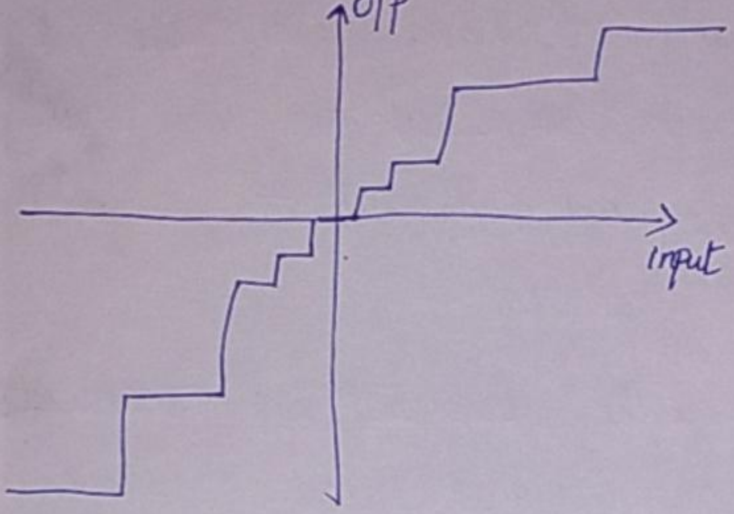
Quantization Noise

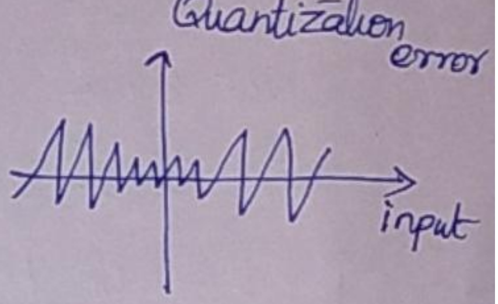
Quantization noise is produced in the transmitter end of a PCM system by rounding off samples values of an analog signal to the nearest representation level of the Quantizer.

The power spectral density of Quantization noise in the receiver o/p is independent of that of the baseband signal and also the power spectral density of Quantization noise has a large BW compared with the signal BW. Thus the effect of Quantization noise is similar to the effect of thermal Noise.

Non Uniform Quantization

In non-uniform Quantization the step Size Δ is not equal over the transfer characteristic of the Quantizer.





Need for Non uniform Quantization

In the transmission of speech signals, the quantizer has to accomodate signals with widely varying power levels. For example, the range of voltages covered by a speech signal is of the order of 1000 to 1. In uniform Quantizer is applied to such a speech signal, then there may be information loss at the low level of i/p signal. To overcome this Quantization with small step size can be performed at low level input. This results in Non uniform Quantization.

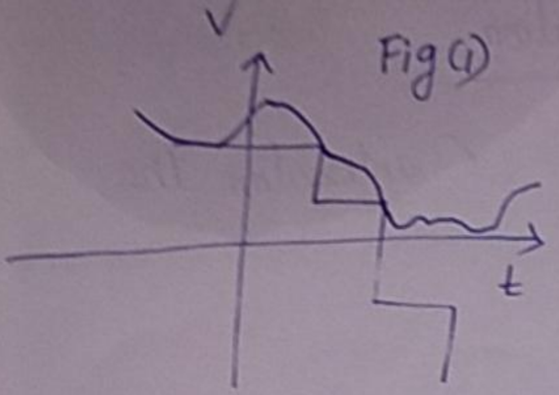


Fig (1)

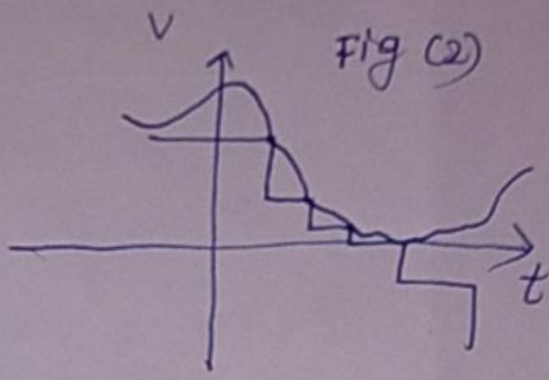
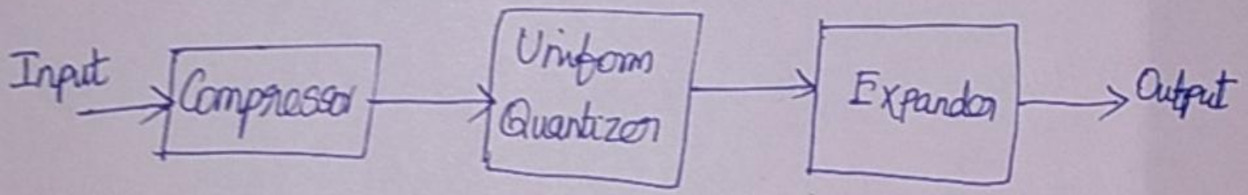


Fig (2)

In fig (1) with uniform quantization, only one representation level is used at the low level of input signal which is not sufficient to reconstruct the signal at the receiver.

In fig (2) with non uniform quantization, weak passages are assigned more representation levels. As a result reconstruction is made easier and information loss is prevented.

Logarithmic Companding



Non uniform Quantization can be achieved by using a Compressor followed by a uniform quantizer.

The Compressor amplifies the signal at low amplitude levels and attenuates the signal at high amplitude levels. After this process uniform Quantization is used. At the receiver side an expander is used to do the reverse process of the Compressor.

The Combination of a Compressor and expander is called a Compander.

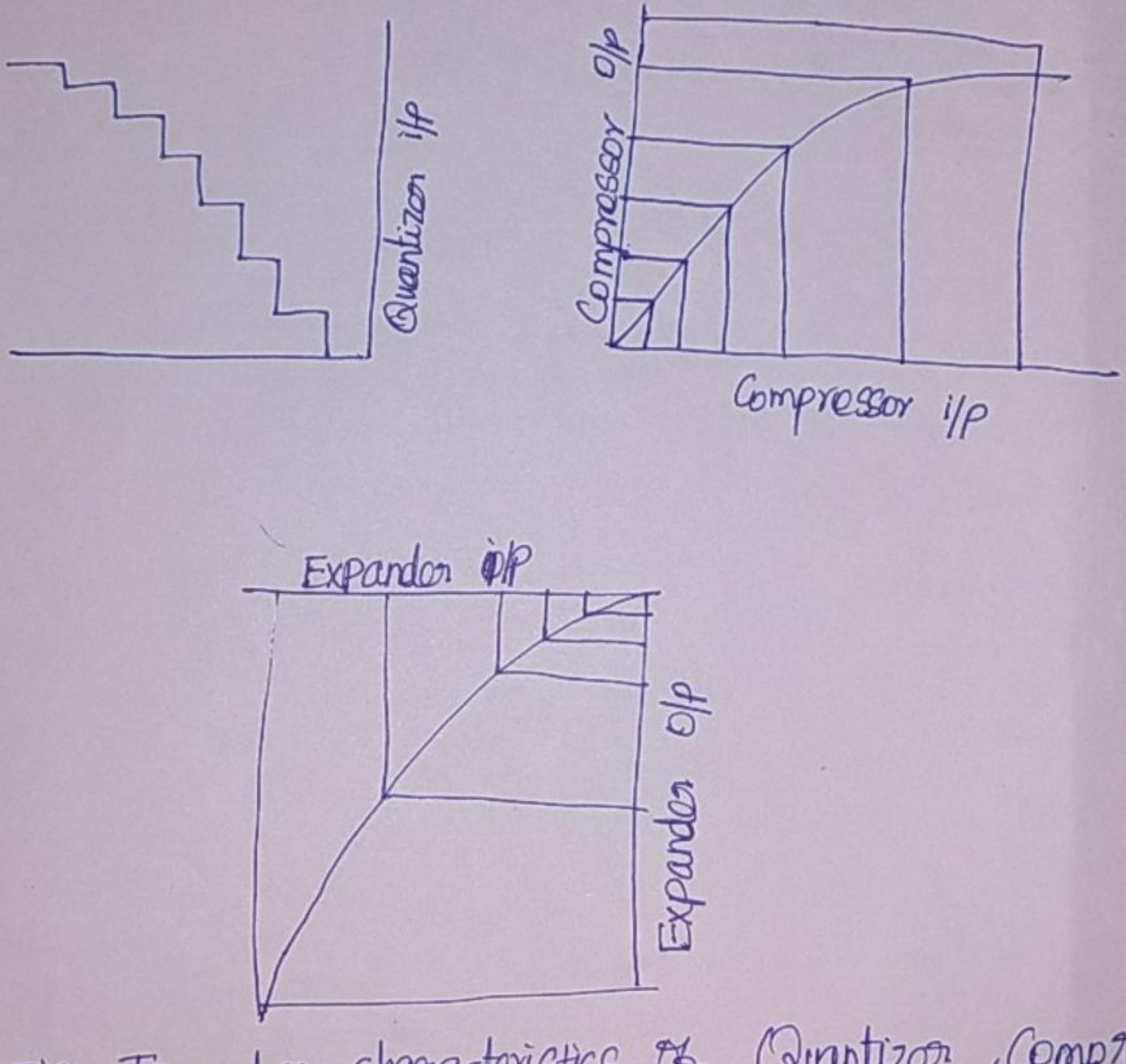
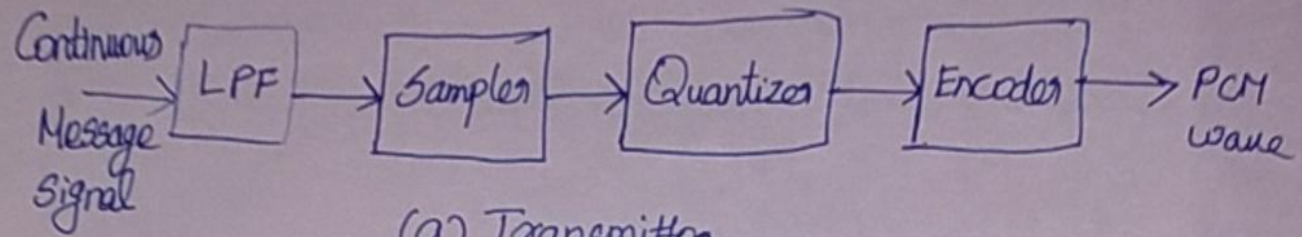
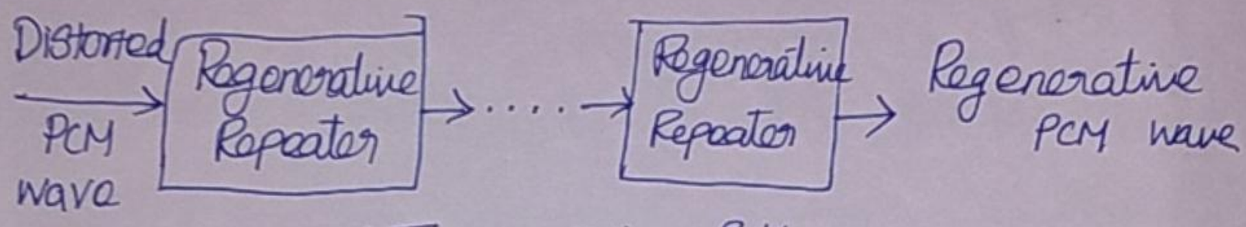


Fig:- Transfer characteristics of Quantizer, Compressor & Expander.

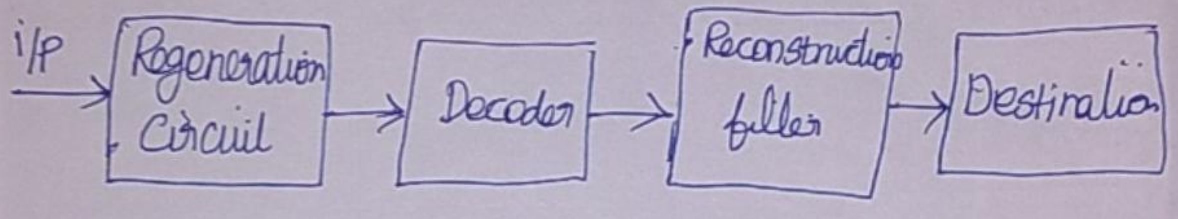
PCM [Pulse Code Modulation]



(a) Transmitter



(b) Transmission Path



(c) Receiver

Pulse code Modulation is a type of signal encoding technique in which the analog information signal is sampled and Quantized, So that both amplitude and time are represented in discrete form.

The elements of PCM system are Shown in fig.

1) Sampling

The incoming message wave is sampled with a train of narrow rectangular pulses with sampling rate greater than twice the highest frequency component W

$$f_s \geq 2W$$

A low pass pre-alias filter is used at the front end of the sampler. Thus sampling converts continuously varying message wave to a limited no. of discrete values per second.

2) Quantizing

Refer Page No. 12

3) Encoding

Encoding process translates the discrete set of values to a more appropriate form of signal which can be easily transmitted over a line, radio path or optical fibre.

Any plan for representing each member of this discrete set of values as a particular arrangements of discrete event is called a code.

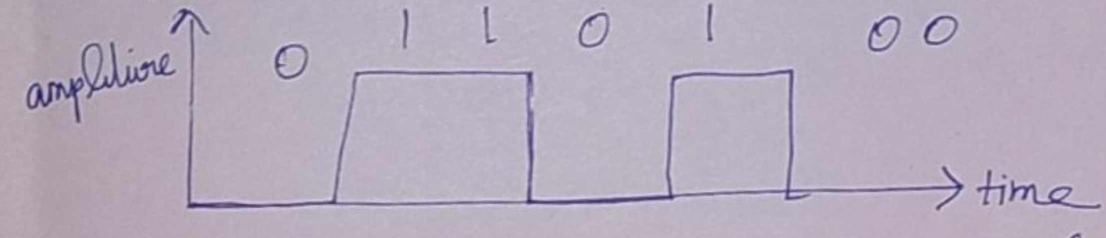
One of the discrete events in a code is called a code element or symbol. Arrangement of symbols used in a code to represent a single value is called code word a character.

In binary code, each symbol may be either of two distinct values. In ternary code, each symbol may be one of 3 distinct values.

Suppose a binary code has code-word with n bits, then 2^n distinct values can be represented by that code.

for ex: sample quantized into 16-levels can be represented by 4-bit code word.

There are several ways to represent the binary code words as waveforms. figure shows examples.

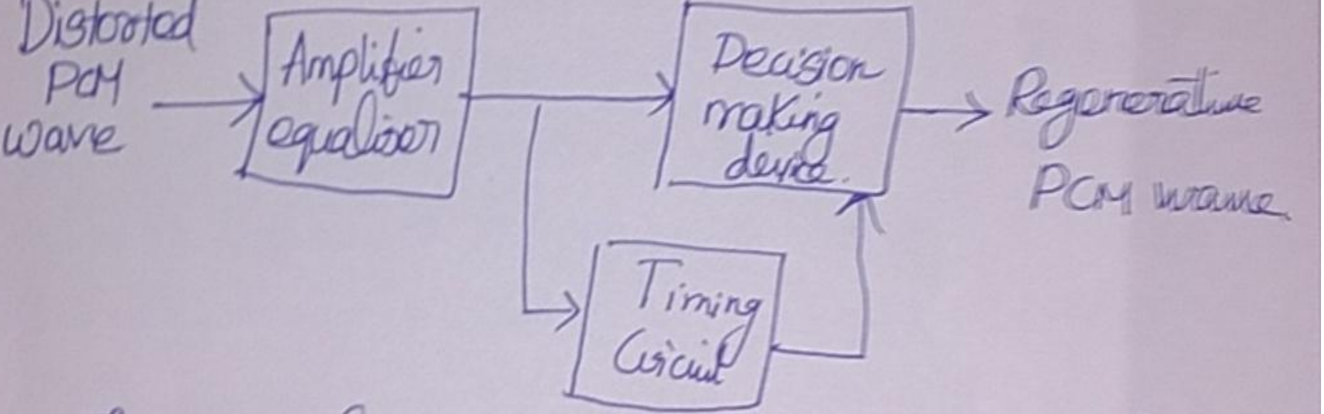


Non return to zero unipolar signal.

4) Regeneration

When PCM wave is transmitted through the channel, it is affected by distortions and noise of the channel. The effect of this distortion and noise are controlled by using chain of regenerative repeaters

A regenerative repeater performs 3 basic functions, namely equalization, timing and decision making.



The equalizer shapes the received pulses so as to compensate effects of amplitude and phase distortions caused by imperfections of the channel

The timing circuit provides a periodic pulse train for sampling the equalized pulses at time when SNR is a maximum.

The decision device is enabled when the amplitude of equalized pulse plus noise exceeds a predetermined threshold device.

However the regenerated signal differs from the original signal due to two main reasons.

1) The presence of channel noise and interference causes the repeater to make wrong decisions and introduces bit errors.

2) If the spacing between received pulses deviates from its assigned values a jitter is introduced.

5) Decoding

The first operation in the receiver is to regenerate the received pulses. These pulses are regrouped into code-words and decoded into a quantized PAM signal. The decoded pulse amplitude is sum of all pulses in the code word weighted by its place value.

6) Reconstruction

The final operation in the receiver is to recover the analog signal by passing decoder o/p through a reconstruction LPF whose cut off frequency is equal to the message Bw 'W'.

7) Multiplexing and synchronization

In applications using PCM, multiplexing can be achieved, which needs synchronization between transmitter and receiver.

B.w Of PCM

Signaling rate Of PCM $\leq f_b = n \times f_s$

$n \rightarrow$ no. of bits/sample $\quad f_s \rightarrow$ sampling rate

B.w of PCM $\geq \frac{1}{2} \times f_b$

$B.w \geq \frac{1}{2} \times n \times f_s$

Coding efficiency

$$\eta_{PCM} = \frac{\text{Maximum no. of bits}}{\text{Actual No. of bits}} \times 100$$

Transmission speed

It is defined as the digital transmission data rate at which serial PCM bits are clocked out of the transmitter.

Application of PCM

* Digital telephone system
* Digital Audio
* Digital video
* PSTN - Public switched telephone n/w

Advantages

* High Noise immunity
* Supports use of Repeaters
* Coding provides high security

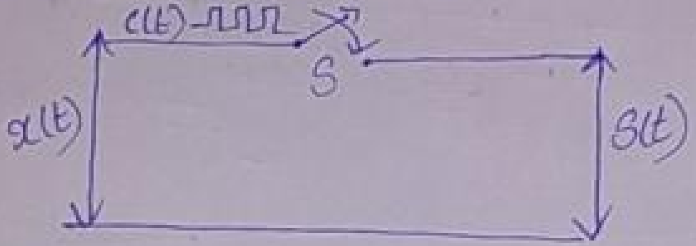
Disadvantages

* Encoding, decoding & Quantization Circuit a Complex.
* PCM requires large B.W

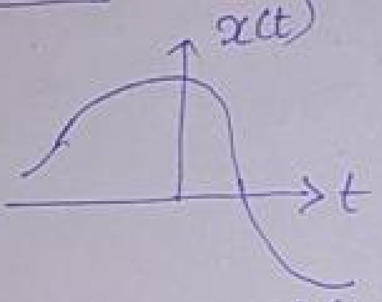
Pulse Amplitude Modulation

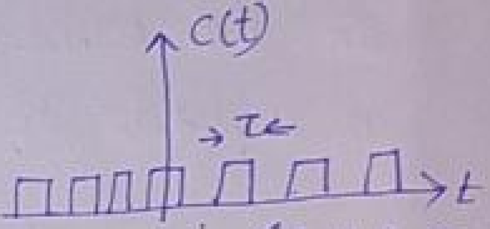
The amplitude of a carrier consisting of a periodic train of rectangular pulse is varied is proportion in proporion to ~~the~~ sample value of a message signal.

The pulses in PAM can be of rectangular shape or the top of the pulse can have variations as per the signal.

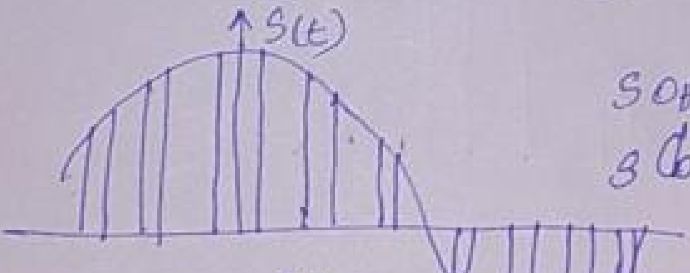


Waveforms.





Carrier (Switch on-off)



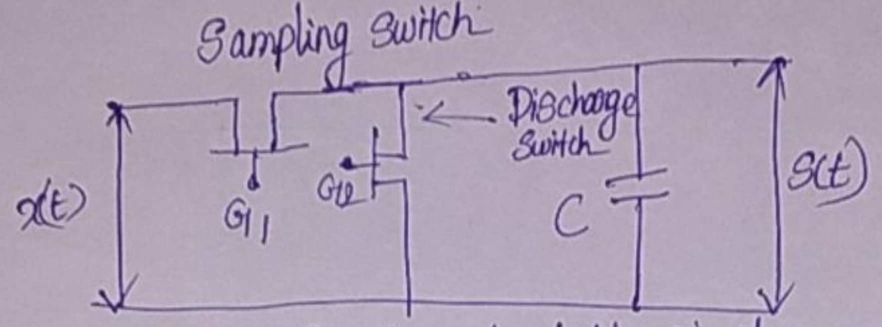
S open → s(t) (low)
S closed → s(t) (high)

PAM.

Let x(t) is a continuous signal. It is sampled at the interval of Ts to generate PAM signal.

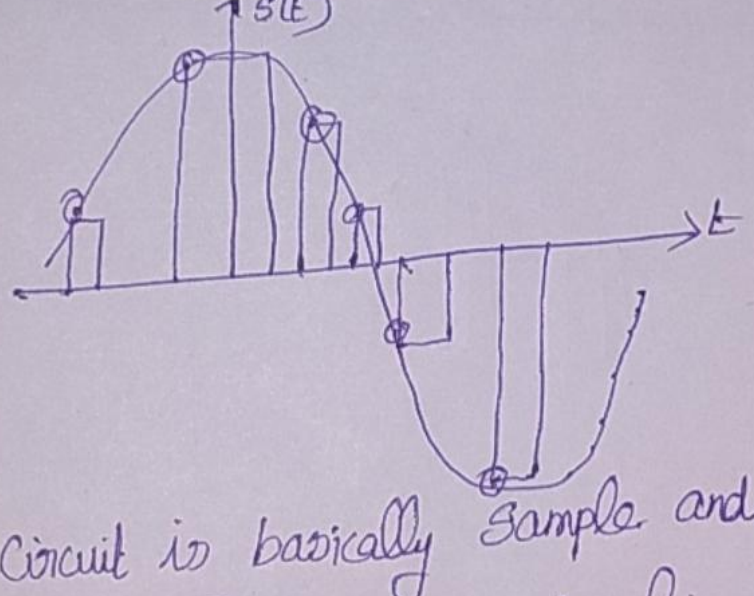
The amplitude of the pulse is same as amplitude of $x(t)$ at the instant of sampling.

The switch 's' can be transmittor (or) FET. The signal $c(t)$ is the base drive of the switch. $s(t)$ is the PAM signal. The width of the pulse in $c(t)$ determines the width of the PAM pulse.



Sample and hold circuit

In this method, the amplitude of the pulse is same as amplitude of input signal. But its top is flat.



The Circuit is basically Sample and hold circuit. At the sampling instant, sampling switch is closed for a very small period. During this period the capacitor c voltage becomes equal to the voltage of x(t).

The sampling switch is opened and capacitor c holds the charge.

The discharge switch s is then closed to discharge capacitor to zero volts.

The discharge switch is then opened and capacitor has no voltage. The capacitor remains charged for a fixed period τ. Thus the flat top sampled PAM signal is generated.

Again after the period Ts. sampling switch is closed to take new sample. This periodic gating of sample and hold circuit generates the flat top PAM signal.

Naturally sampled PAM signal

$$s(t) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} x(t) \, \text{sinc}(f_n \tau) \, e^{j2\pi n f_s t}$$

Spectrum of Naturally sampled PAM signal

$$S(f) = \frac{\tau A}{T_s} \sum_{n=\infty}^{\infty} \text{sinc}(n f_s \tau) \times (f - n f_s)$$

Flat top PAM

$$S(t) = \sum_{n=-\infty}^{\infty} x(nTs)\, h(t-nTs)$$

Spectrum of Flat top PAM

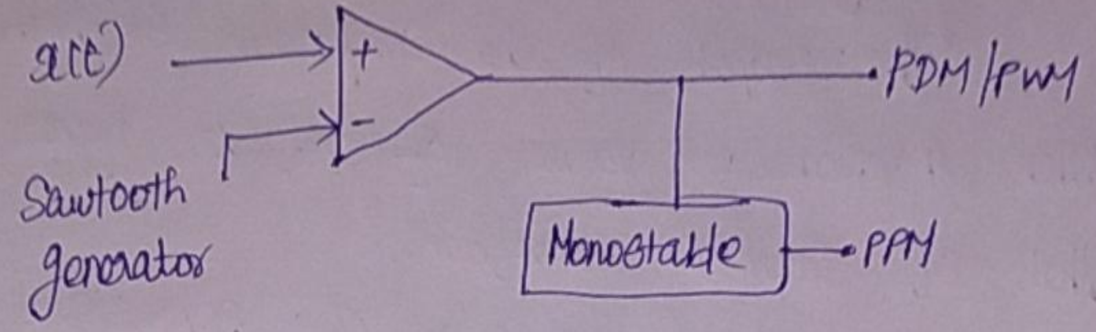
$$S(f) = fs \sum_{n=-\infty}^{\infty} x(f-nfs)\, H(f)$$

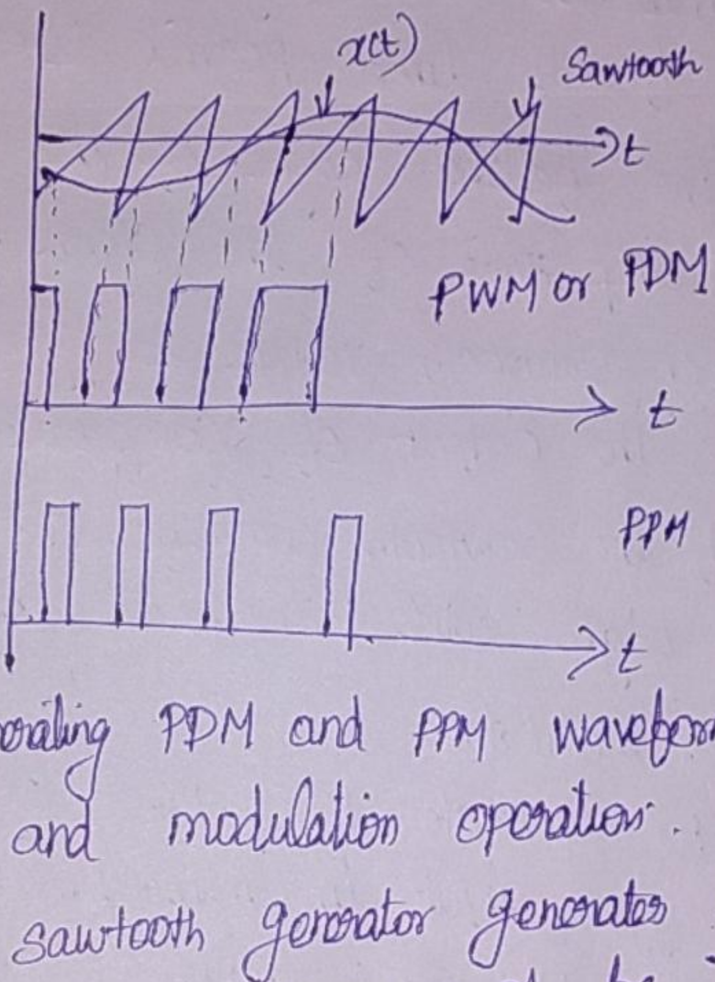
Pulse width and Pulse position Modulation

- Both Modulate the time parameter of pulses.
- PPM has fixed width pulses where as width of PPM pulse varies.
- with PPM, the position of a Constant width pulse within a prescribed time slot is varied according to the amplitude of the sample of the analog signal.

In PDM (or) PWM, the width of the Constant amplitude pulse is varied proportional to the amplitude of analog signal at the time signal is Sampled.

Block diagram.





* for generating PDM and PPM waveform we need sampling and modulation operation.

The sawtooth generator generates the Sawtooth signal with a frequency of f_s. This Sawtooth signal is also called as sampling signal. and it is applied to the inverting input of Comparator.

The modulating signal x(t) is applied to the non inverting input of the comparator.

The output of the comparator is high only when instantaneous value of x(t) is higher than that of sawtooth waveform.

Thus the leading edge of PPM signal occurs at the fixed time period (i.e) KTs.

The falling edge of the PDM wave depends on the amplitude of the signal x(t).

When sawtooth voltage is greater than voltage of x(t) the output of comparator remains zero.

If the sawtooth waveform is reversed, then falling edge will be fixed and leading edge will be modulated.

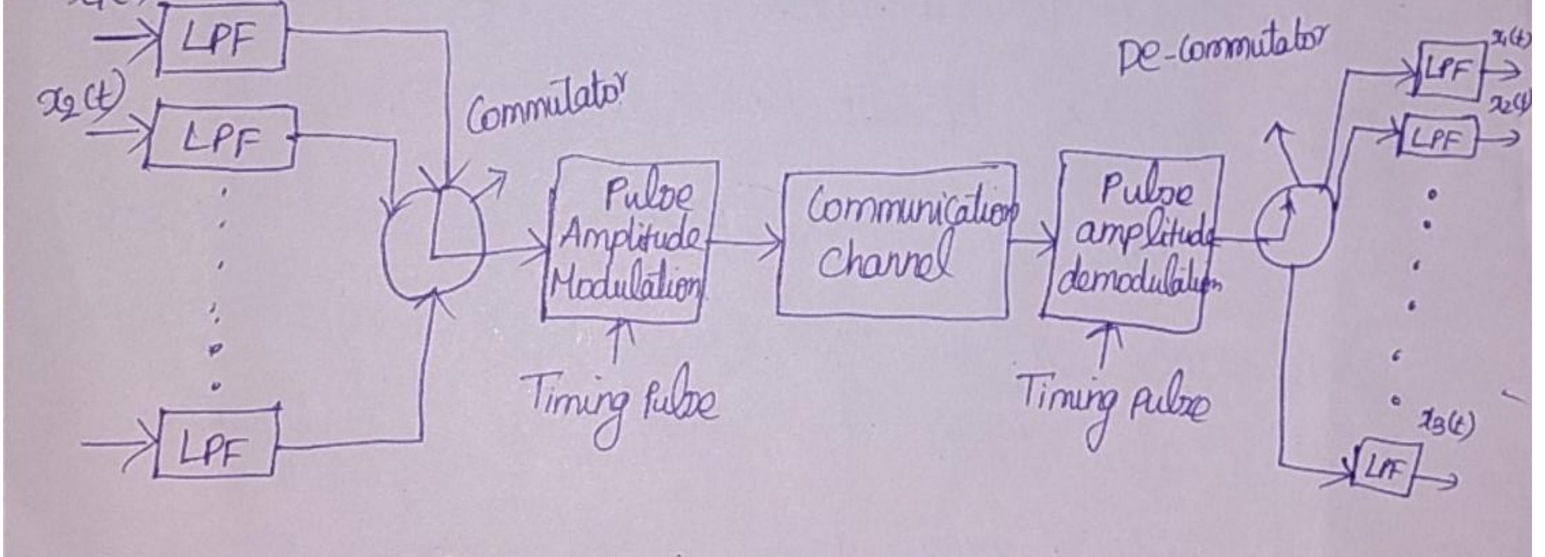
To generate PPM, PDM signal is used as the trigger input to monostable multivibrator. The monostable O/P remains zero until it is triggered. The monostable is triggered on the falling edge of PDM.

The width of the PPM pulse is determined by the monostable multivibrator.

TDM (Time Division Multiplexing)

The transmission of the message samples utilizes the communication channel for only a fraction of the sampling interval on a periodic basis, and in this way the time interval between adjacent samples is cleared for use by some other independent message sources on a time shared basis.

. In Time division Multiplexing, all the signals to be transmitted are not transmitted simultaneously. Instead they are transmitted one by one, so each signal will be transmitted for very short time.



Block diagram of TDM System

Each input message signal is first band limited by a low pass anti aliasing filter to remove the frequencies, that are non essential to an adequate signal representation.

Then the low pass filter outputs are applied to a Commutator, which is usually implemented using rotating Switch or electronic Switch. It rotates at fs rotations per second. As the switch rotates, it is going to make Contact with the position 1,2,3 or N for Short time.

Hence these Switch arm will connect these N Signals one by one to the Communication channel. The rotating Switch is samples each message during each of its rotations

The function of the Commutator is two fold.

(i) To take a narrow Sample of each N input messages at a rate fs that is slightly higher than 2fm, Where fm is the cut off frequency of the anti aliasing filter

(ii) To sequentially interleave these N samples inside

As per the rotation of the Commutator of the Samples of the data inputs are Collected by it. Here fs is the rate of rotation of the Commutator, thus denotes Sampling frequency of the system.

Suppose we have n data inputs, then one after the other, according to the rotation, these data inputs after getting multiplexed transmitted over the Common channel.

Now at the receiver end, a de commutator is placed that is Synchronized with the Commutator at the transmitting end. This de Commutator separates the time division multiplexed signal at the received end.

The Commutator and de commutator must have Same rotational Speed So as to have accurate demultiplexing of the signal at receiving end.

According to the rotation performed by the de~~modulator~~ Commutator, the Samples are collected by the LPF and the original data input is recovered at the receiver.

Let fm be the maximum signal frequency and fs is the sampling frequency then

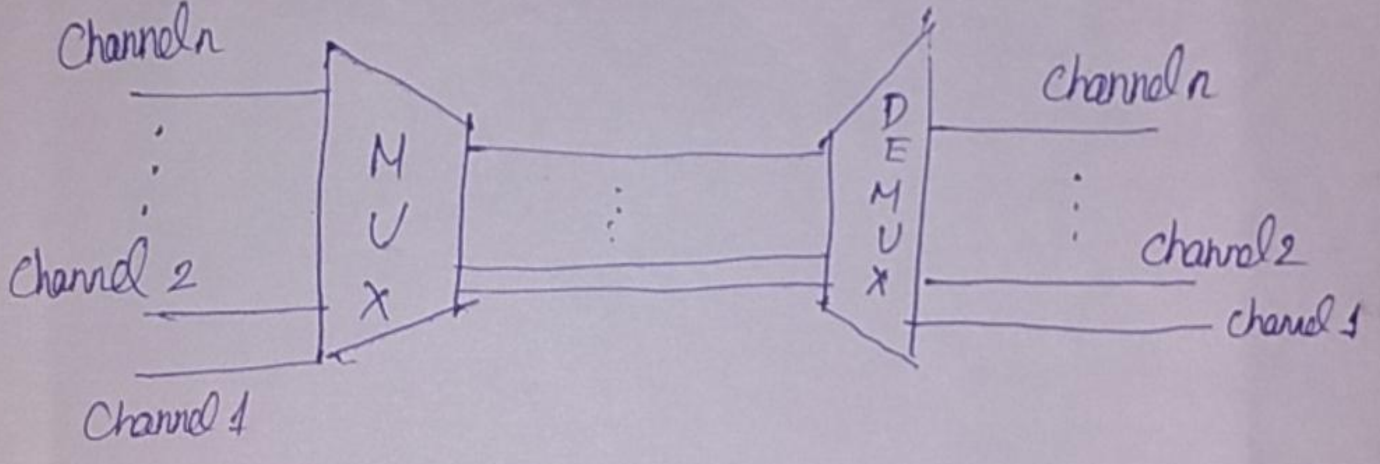
$$f_s \geq 2f_m$$

Thus, the time duration in between successive sample is given as

$$T_s = 1/f_s$$

Frequency division Multiplexing

In this, a number of signals are transmitted at the same time, and each source transfer its signals in the allotted frequency range. There is a suitable frequency gap between the adjacent signals to avoid over lapping. Since the signals are transmitted in the allotted frequencies so this decreases the probability of collision.



Application of FDM

1) In the first generation of mobile phones, FDM was used.

2. The use of FDM in television broadcasting.

3. FDM is used to broadcast FM and AM radio frequencies